UNIVERSITY OF VICTORIA

Midterm

July 26, 2017

solutions

NAME: ______________________________	
STUDENT NUMBER: V00______________	

Course Name & No.	**Statistical Inference Economics 246**
Section(s)	**A01**
CRN:	**31214**
Instructor:	**Betty Johnson**
Duration:	**1hour 50 minutes**

This exam has a total of _10_ pages including this cover page.
Students must count the number of pages and report any discrepancy immediately to the Invigilator.

This exam is to be answered: In Booklets provided

Marking Scheme:

Part I:
- Q1: 20 marks
- Q2: 8 marks
- Q3: 8 marks

Part II:
- Q4: 10 marks
- Q5: 10 marks
- Q6: 10 marks

Part III:
- Q7: 10 marks

Part IV:
- Q8: 9 marks
- Q9: 12 marks
- Q10: 3 marks

Materials allowed: Non-programmable calculator

Part I: Multiple choice

1) In a recent survey of high school students, it was found that the average amount of money spent on entertainment each week was normally distributed with a mean of $52.30 and a standard deviation of $18.23. Assuming these values are representative of all high school students, what is the probability that for a sample of 25, the average amount spent by each student exceeds $60?

A) 0.3372
B) 0.0174
C) 0.1628
D) 0.4826

Answer: B

2) If a sample of size 100 is taken from a population whose standard deviation is equal to 100, then the standard error of the mean is equal to:

A) 10
B) 100
C) 1,000
D) 10,000

Answer: A

3) What is the name of the parameter that determines the shape of the chi-square distribution?

A) mean
B) variance
C) proportion
D) degrees of freedom

Answer: D

4) If all possible samples of size *n* are drawn from an infinite population with a mean of 20 and a standard deviation of 5, then the standard error of the sampling distribution of sample means is equal to 1.0 only for samples of size:

A) 5
B) 15
C) 20
D) 25

Answer: D

5) Why is the central limit theorem important in statistics?

A) Because for a large sample size *n*, it says the population is approximately normal.
B) Because for any population, it says the sampling distribution of the sample mean is approximately normal, regardless of the shape of the population.
C) Because for a large sample size *n*, it says the sampling distribution of the sample mean is approximately normal, regardless of the shape of the population.
D) Because for any sample size *n*, it says the sampling distribution of the sample mean is approximately normal.

Answer: C

6) The average score of all students who took a particular statistics class last semester has a mean of 70 and a standard deviation of 3.0. Suppose 36 students who are taking the class this semester are selected at random. Find the probability that the average score of the 36 students exceeds 71.

A) 0.0228
B) 0.0772
C) 0.1228
D) 0.1772.

Answer: A

7) The amount of material used in making a custom sail for a sailboat is normally distributed with a standard deviation of 64 square feet. For a random sample of 15 sails, the mean amount of material used is 912 square feet. Which of the following represents a 99% confidence interval for the population mean amount of material used in a custom sail?

A) 912 ± 49.2
B) 912 ± 42.6
C) 912 ± 44.3
D) 912 ± 46.8

Answer: B

8) Which of the following statements is true regarding the width of a confidence interval for a population proportion?

A) It is narrower for 95% confidence than for 90% confidence.
B) It is wider for a sample of size 80 than for a sample of size 40.
C) It is wider for 95% confidence than for 99% confidence.
D) It is narrower when the sample proportion is 0.20 than when the sample proportion is 0.50.

Answer: D

9) Which of the following distributions is used when estimating the population mean from a normal population with unknown variance?

A) the t distribution with $n + 1$ degrees of freedom
B) the t distribution with n degrees of freedom
C) the t distribution with $n - 1$ degrees of freedom
D) the t distribution with $2n$ degrees of freedom

Answer: C

10) If a sample has 20 observations and a 90% confidence estimate for μ is needed, the appropriate t-score is:

A) 2.120
B) 1.746
C) 2.131
D) 1.729

Answer: D

11) If a sample of size 30 is selected, the value of A for the probability $P(t \geq A) = 0.01$ is:

A) 2.247
B) 2.045
C) 2.462
D) 2.750

Answer: C

12) A random sample of size 15 is taken from a normally distributed population with a sample mean of 75 and a sample variance of 25. The upper limit of a 95% confidence interval for the population mean is equal to:

A) 77.530
B) 72.231
C) 74.727
D) 79.273

Answer: A

13) If a sample of size 81 is taken from a population whose standard deviation is equal to 81, then the standard error of the mean is equal to

A) 9
B) 27
C) 1
D) None of the above

ANSWER: A

14) If a random sample of size n is drawn from a normal population, then the sampling distribution of sample means will be:

A) normal for all values of n
B) normal only for $n > 30$
C) approximately normal for all values of n
D) approximately normal only for $n > 30$

ANSWER: A

15) If the standard deviation of the sampling distribution of sample means is 7.0 for samples of size 64, then the population standard deviation must be

A) 56
B) 448.
C) 3136
D) 7.

ANSWER: A

16) Let X_1, X_2, X_3, and X_4 be a random sample of observations from a population with mean μ and variance σ^2. Consider the following estimator of μ: $\hat{\theta}_1 = 0.6\ X_1 + 0.4\ X_2 + 0.25\ X_3 + 0.45\ X_4$. What is the variance of $\hat{\theta}_1$?

A) 1.700 σ^2

B) 0.785 σ^2

C) 2.890 σ^2

D) 0.425 σ^2

Answer: B

17) If a sample of size 41 is selected, the value of *A* for the probability $P(-A \leq t \leq A) = 0.90$ is:

A) 1.303

B) 1.684

C) 2.021

D) 2.423

Answer: B

18) Which of the following statements is correct?

A) A point estimate is an estimate of the range of a population parameter

B) A point estimate is a single value estimate of the value of a population parameter

C) A point estimate is an unbiased estimator if its standard deviation is the same as the actual value of the population standard deviation

D) All of the above

ANSWER: B

19) The bias of an unbiased estimator is equal to:

A) 1

B) 0

C) Infinity

D) Depends on the parameters of the question.

Answer: B

20) The larger the level of confidence (e.g., .99 versus .95) used in constructing a confidence interval estimate of the population mean, the:

A) larger the probability that the confidence interval will contain the population mean

B) the larger the sample size

C) smaller the value of $z_{\alpha/2}$

D) narrower the confidence interval

ANSWER: A

Question 2: (8 marks)

The length of time it takes to fill an order at a local Tim Hortons is normally distributed with a mean of 2.4 minutes and a standard deviation of 1.5 minutes.

a) What is the probability that the average waiting time for a random sample of 25 customers is between 1.7 and 2.9 minutes?

ANSWER:

$P(1.7< \bar{X} <2.9) = P(Z < 1.67) – P(Z < -2.33) = .9525-(1-.9901)=0.9426$

b) The probability is 95% that the average waiting time for a random sample of twenty five customers is greater than how many minutes?

ANSWER:

$$P(\bar{X} > a) = (0.95) \Rightarrow P\left(Z > \frac{a-2.4}{1.5/\sqrt{25}} \right) = P\left(Z > \frac{a-2.4}{0.3} \right) = 0.95$$

$$\Rightarrow \frac{a-2.4}{0.3} = -1.645 \Rightarrow a = 1.9065 \text{ minutes}$$

Question 3: (8 Marks)

The filling machine at a local McDonald's is operating correctly when the variance of the fill amount is equal to 0.8 ounces. Assume that the fill amounts follow a normal distribution.

a) What is the probability that for a sample of 26 bottles, the sample variance is greater than 0.5?

ANSWER:

$$P(s^2 > 0.5) = P\left[\frac{(n-1)s^2}{\sigma^2} > \frac{(25)(0.5)}{0.8} \right] = P(\chi^2_{25} > 15.625) \text{ between } 90\% \text{ to } 95\%$$

b) The probability is 0.10 that for a sample of 26 bottles, the sample variance is less than what number? (Determine the value of the sample variance that would give the probability 10% or less.)

$$P(s^2 < k) = 0.1 \Rightarrow P\left[\frac{(n-1)s^2}{\sigma^2} < \frac{25k}{0.8} \right] = 0.10$$

$$Critical\ value = 16.473$$

$$\frac{25S^2}{0.8} = 16.473$$

$$\Rightarrow 83.33s^2 = 16.47$$

$$\Rightarrow s^2 = 0.5271$$

Part II: Concepts

Question 4: Describe the concept of stratified sampling. Illustrate the technique with an example.

Total marks: 10

There are two random sampling techniques that use prior knowledge about the population and hence, reduce the costs of simple random sampling:

1) **Stratified Sampling**
2) **Cluster Sampling**

(I) Stratified Sampling

This technique divides the population into a number of d**istinct** and **similar** subgroups, and then selects a proportionate number of items from each subgroup.

"The use of stratified sampling requires that a population be divided into homogeneous groups called strata. Each stratum is then sampled according to certain specified criteria."

➢If we can identify certain **characteristics** in the population and can separate these characteristics into subgroups, we require fewer sample points to determine the level or concentration of the characteristic under study.

➢The optimal method of selecting strata is to find groups with a large variability between strata, but with only a small variability within the strata.

i.e. Groups should have large ***inter-strata variation***, but little ***intra-strata variation.***

Each subgroup contains persons who share **common traits** and each subgroup is distinctly different.

Advantages:

(1) If homogeneous subsets of a population can be identified, then only a relatively small number of sample observations are needed to determine the characteristics of each subset. Thus, stratified sampling is usually less expensive than simple random sampling, because we only require a small number of sample points to get an **accurate** measure of the characteristic under study.

(2) Use of prior knowledge about the population may improve the accuracy of the statistical inference based on stratified sampling as compared to simple random sampling

➔ improvement in the "efficiency" of the estimate.

Example: A politician hires a company to determine the political platform he/she should stress: job creation or lower taxes. The research team divides the population into income classes: Upper, middle and lower income groups. Then, they sample the proportionate amount from each strata:

- In this case, we know the population is composed of 15% upper income, 55% middle income and 30% lower income. We then take a sample that contains 15% from the upper income strata, 55% from the middle income strata, and 30% from the lower income strata.

Question 5: Describe the concept of efficiency with respect to estimator properties. ***Total Marks: 10***

(I) **Efficiency:** The most efficient estimator among a group of unbiased estimators is the one with the smallest variance (or dispersion of values).

If $\hat{\theta}$ and $\tilde{\theta}$ are both unbiased estimators of θ, and the variance of $\hat{\theta}$ is less than or equal to the variance of $\tilde{\theta}$, $V(\hat{\theta}) \leq V(\tilde{\theta})$, then $\hat{\theta}$ is an efficient estimator of θ, relative to $\tilde{\theta}$.

The most efficient estimator is called the **best unbiased estimator**, where **"best"** implies **minimum variance**.

Relative Efficiency: is defined as the ratio of the variance of the two estimators:

$$\text{Relative Efficiency} = \frac{V(\hat{\theta})}{V(\tilde{\theta})} = \frac{\text{Variance of the first estimator}}{\text{Variance of the second estimator}};$$

R.E. < 1 if $\hat{\theta}$ is efficient relative to $\tilde{\theta}$.

R.E. > 1 if $\tilde{\theta}$ is efficient relative to $\hat{\theta}$.

R.E. = 1 equally efficient.

What if one or both of the estimators are biased?

We then calculate the MSE of the estimators to determine which estimator is best:

Definition: Mean Squared Error (MSE)

The mean squared error (MSE) of $\hat{\theta}$ is $E(\hat{\theta}-\theta)^2$:

$$MSE(\hat{\theta}) = E(\hat{\theta}-\theta)^2 \quad \Rightarrow add \ \& \ subtract \ E(\hat{\theta})$$

$$= E\left[\hat{\theta} - E(\hat{\theta}) + E(\hat{\theta}) - \theta\right]^2 \Rightarrow group \ in \ two$$

$$= E\left[\left(\hat{\theta} - E(\hat{\theta})\right) + \left(E(\hat{\theta}) - \theta\right)\right]^2 \quad \Rightarrow expand$$

$$= \underbrace{E\left[(\hat{\theta} - E(\hat{\theta}))^2\right]}_{Variance(\hat{\theta})} + \underbrace{E\left[(E(\hat{\theta}) - \theta)^2\right]}_{Bias(\hat{\theta})} + \underbrace{2E\left((\hat{\theta} - E(\hat{\theta}))\right)\left((E(\hat{\theta}) - \theta)\right)}_{=0}$$

$$= \quad V(\hat{\theta}) \quad + \quad \left[(Bias(\hat{\theta})\right]^2 \quad + \quad [0]$$

The MSE enables us to compare <u>biased</u> estimators.
<u>Note:</u>

$$MSE(\hat{\theta}) = V(\hat{\theta}) \ if \ \ the \ Bias(\hat{\theta}) = 0$$

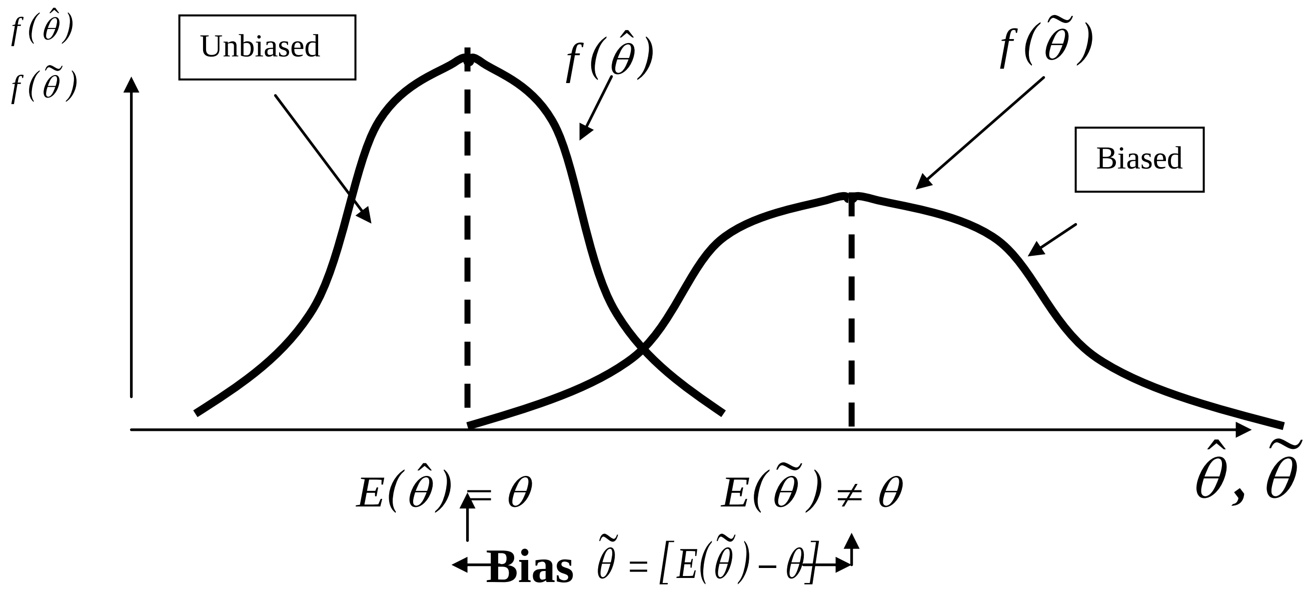
.

<u>Definition</u>:

Let $\hat{\theta}$ and $\tilde{\theta}$ be two estimators of θ .

Then $\hat{\theta}$ is an efficient compared to $\tilde{\theta}$ if MSE ($\hat{\theta}$) $\leq$ MSE ($\tilde{\theta}$).

Graphical illustration: (I)



In this example, the $V(\hat{\theta}) < V(\tilde{\theta})$, and $\hat{\theta}$ is unbiased while $\tilde{\theta}$ is biased.

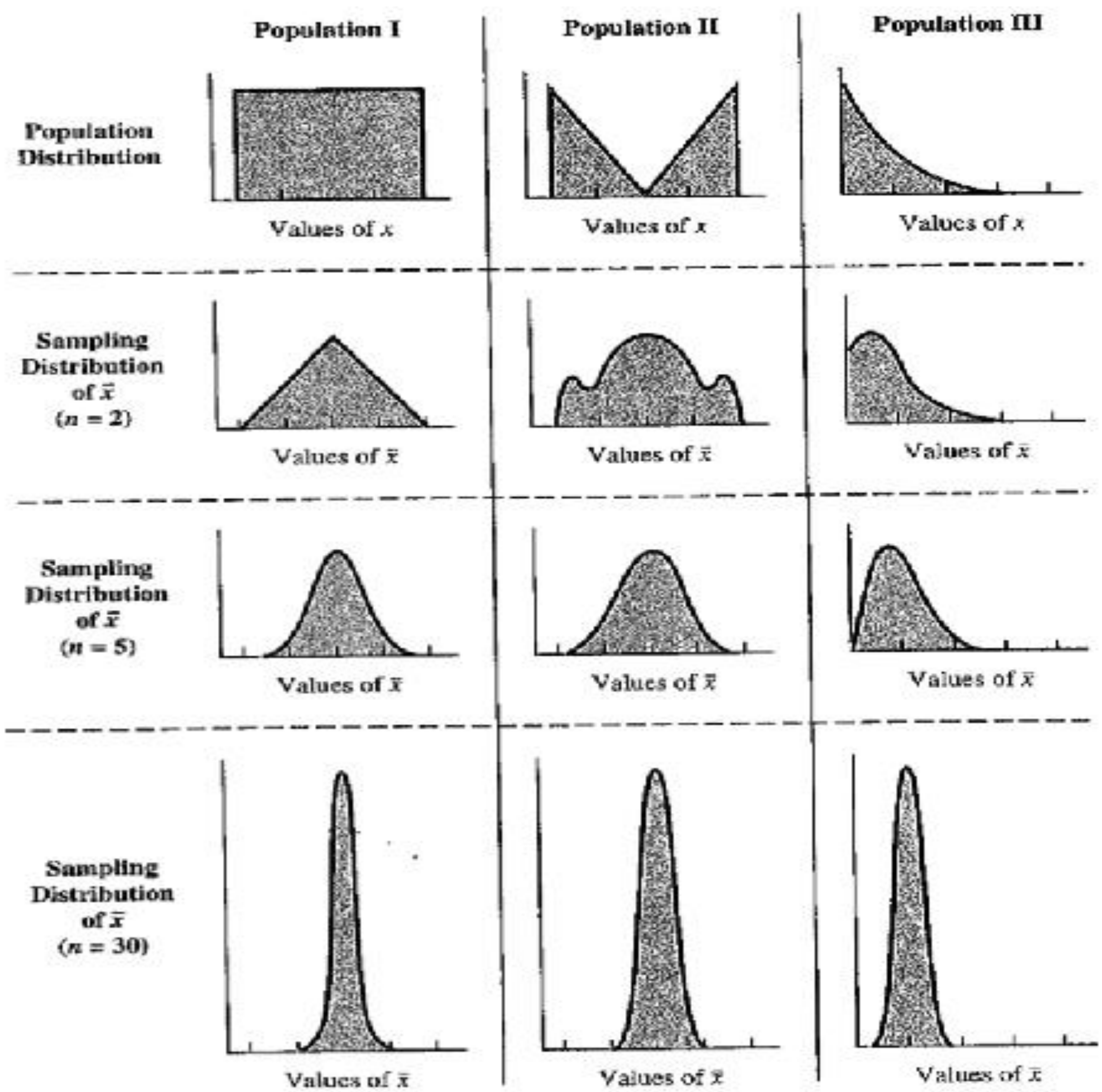
Question 6: Define and illustrate the concept of Central Limit Theorem. ***Total Marks: 10***

Central Limit Theorem:

"Regardless of the distribution of the parent population, as long as it has a finite mean μ and variance σ^2, the distribution of the means of the random samples will approach a normal distribution, with mean μ and variance σ^2/n, as the sample size n, goes to infinity."

(I) When the parent population is normal, the sampling distribution of $\overline{X}$ is **exactly** normal.

(II) When the parent population is not normal or unknown, the sampling distribution of $\overline{X}$ is **approximately** normal as the sample size increases.



Part III: Proofs

Question 7*: Total marks: 10*

(i) Using the fact that the mean of the chi-squared distribution is (n-1), prove that $E(S^2) = \sigma^2$

$E\left(s^2\right) = \sigma^2$

Since $E(\chi^2) = n-1$

and $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

if you take the expectation:

$$\mathrm{E}\left[\frac{(n-1)s^2}{\sigma^2}\right] = n-1$$

$$E\left[s^2\right] = \frac{n-1}{n-1}\sigma^2$$

$$E\left[s^2\right] = \sigma^2$$

(ii) Using the fact that the variance of the chi-squared distribution is 2(n-1), prove that $V(s^2) = \dfrac{2\sigma^4}{n-1}$ ..

$$V(\chi^2) = 2(n-1)$$

$$V\left(\frac{(n-1)s^2}{\sigma^2}\right) = 2(n-1)$$

$$\frac{(n-1)^2}{\sigma^4} V(s^2) = 2(n-1)$$

$$V(s^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{(n-1)}$$

Part IV: SHORT ANSWER

Question 8: (9 marks)

Suppose we have two estimators of the population parameter θ:

$$E\left(\hat{\theta}\right) = \theta + 6/n$$
$$V\left(\hat{\theta}\right) = \sigma^2 n^3$$

and

$$E\left(\tilde{\theta}\right) = \theta + 81/n^3$$
$$V\left(\tilde{\theta}\right) = 4\sigma^2 n^5$$

(i) Determine the bias, if any, of each estimator.

$$Bias\left(\hat{\theta}\right) = E(\hat{\theta}) - \theta = \left(\theta + 6/n\right) - \theta = 6/n$$
$$Bias\left(\tilde{\theta}\right) = E(\tilde{\theta}) - \theta = \left(\theta + 81/n^3\right) - \theta = 81/n^3$$

(ii) Determine the MSE. Which estimator is preferred?

$$MSE(\hat{\theta}) = V(\hat{\theta}) + bias(\hat{\theta})^2 = \sigma^2 n^3 + \left(\frac{36}{n^2}\right)$$

$$MSE(\tilde{\theta}) = V(\tilde{\theta}) + bias(\tilde{\theta})^2 = 4\sigma^2 n^5 + \frac{6561}{(n^3)^2}$$

First estimator is preferred.

(iii) Determine if the estimators are consistent. Explain.

The bias and variance go to zero as n goes to infinity if the estimator is consistent. Consistency property illustrates how the sampling distribution of an estimator changes as the sample size changes.

Both estimators are not mean square consistent. The bias goes to zero, but the variance does not go to zero as n approaches infinity.

<u>Question 9: (12 Marks)</u> Consider the following population of data: {12, 13, 14}.

(i) Determine the mean and variance of the population.

Total marks: 4

$$\mu = \frac{1}{3}(12+13+14) = 39/3 = 13$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2 = \frac{1}{N}\left[\sum_{i=1}^{N} X_i^2\right] - \mu^2$$

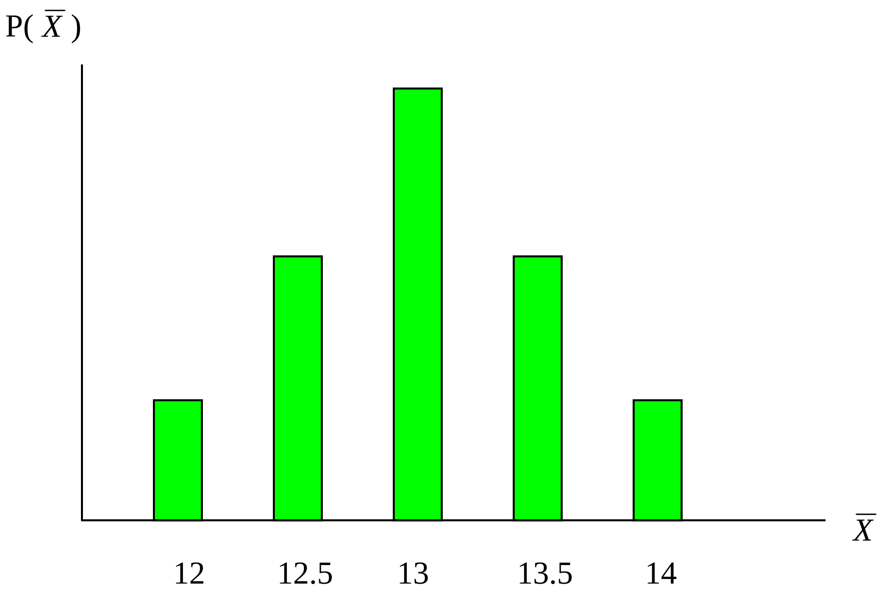
$$\sigma^2 = \frac{1}{3}(509) - 13^2 = 169.6667 - 169 = 0.6667 = \frac{2}{3}$$

(ii) Determine the sampling distribution of the sample mean for a sample of size 2. Graph this distribution with a simple bar graph.

Total marks: 4

X1,X2	$\bar{X}$
12, 12	**12**
12, 13	**12.5**
12, 14	**13**
13, 12	**12.5**
13, 13	**13**
13, 14	**135**
14, 12	**13**
14, 13	**13.5**
14, 14	**14**

$\bar{X}$	**P($\bar{X}$)**
12	**1/9**
12.5	**2/9**
13	**3/9**
13.5	**2/9**
14	**1/9**



(iii) **Determine the variance of $\bar{X}$?** Total Marks:4

0.6667/2=0.3333=1/3

<u>Question 10: True/False and explain. (3 marks)</u>

A narrower confidence interval for a population parameter with a given confidence level can be obtained by increasing the sample size.

True: as more of the population information is used in the sample the narrower the confidence interval.

Width of a confidence interval is $2\times(Z_{\alpha/2}(\sigma/\sqrt{n})$. As n gets larger, the width gets narrower.

End of Exam

Formulae

Central Location:

Population mean $\mu = \frac{1}{N}\left[\sum x_i\right]$

Grouped Population Mean $\mu = \frac{\sum x_i f_i}{\sum f_i} = \frac{1}{N}\left[\sum x_i f_i\right]$

Sample Mean $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Sample Mean for frequency distribution: $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i f_i$

Mean of the Sample Mean ($\overline{X}$) $E(\overline{X}) = \mu_{\overline{X}} = \sum_{i=1}^{k} \overline{X}_i P(\overline{X}_i)$

where: i= 1,2,...,k, and k is the number of distinct possible values of $\overline{X}$.

Dispersion:

Population variance $\sigma^2 = \left(\frac{1}{N}\right)\sum\left(x_i - \mu\right)^2$

(Grouped data $\sigma^2 = \left(\frac{1}{N}\right)\left[\sum\left(x_i - \mu\right)^2 f_i\right]$

Sample variance for frequency distribution: $s^2 = \frac{1}{(n-1)}\sum\left(x_i - \overline{x}\right)^2 f_i$

Sample variance $s^2 = \frac{1}{(n-1)}\sum\left(x_i - \overline{x}\right)^2$

Sample Standard Deviation $s = \sqrt{s^2}$

Variance of the Sample Mean $\sigma_{\overline{X}}^2 = V(\overline{X}) = \sigma^2/n = \sum_{\overline{X}} (\overline{X} - \mu_{\overline{X}})^2 P(\overline{X})$

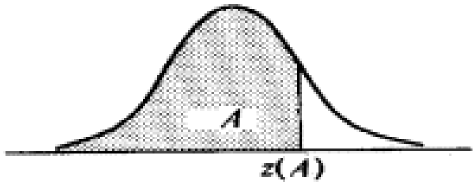
Standard Error of the mean: $\sigma_{\overline{X}} = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$.

Distributions:

Standard Normal: $Z = \frac{(X - \mu)}{\sigma}$; **The Standardization of** $\overline{X}$: $Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$

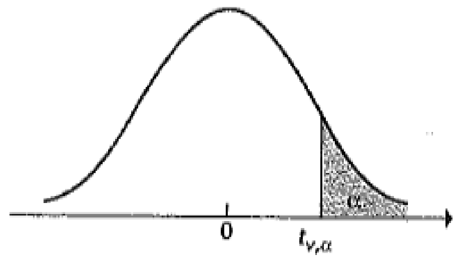
t-distribution $t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$; **Chi-square distribution** $\chi_{n-1}^2 = \frac{\nu(s^2)}{\sigma^2} = \frac{(n-1)s^2}{\sigma^2}$

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

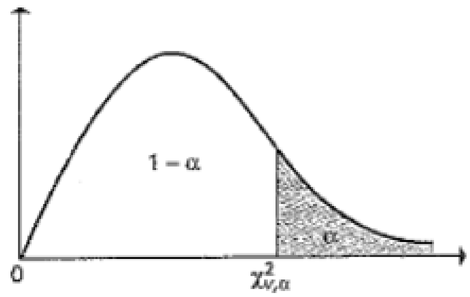
Table 8 Cutoff Points for the Student's t Distribution



For selected probabilities, α, the table shows the values $t_{\nu,\alpha}$ such that $P(t_\nu > t_{\nu,\alpha}) = \alpha$, where t_ν is a Student's t random variable with ν degrees of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

ν	α				
	0.100	0.050	0.025	0.010	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
∞	1.282	1.645	1.960	2.326	2.576

Table 7 Cutoff Points of the Chi-Square Distribution Function



For selected probabilities α, the table shows the values $\chi^2_{v,\alpha}$ such that $P(\chi^2_v > \chi^2_{v,\alpha}) = \alpha$, where χ^2_v is a chi-square random variable with v degrees of freedom. For example, the probability is .100 that a chi-square random variable with 10 degrees of freedom is greater than 15.99.

v	α									
	.995	.990	.975	.950	.900	.100	.050	.025	.010	.005
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2