Introduction to Quantum Computing
Assignment 2 - Due Jan. 29
Operators, measurements, and the Bloch sphere

1. **Eigenvalues and eigenvectors for the Pauli matrices.**– Find the eigenvalues and eigenvectors for the four Pauli matrices: \( \hat{I}, \hat{X}, \hat{Y}, \hat{Z} \).

2. **Operator that can not be diagonalized.**– Consider the following operator

\[
\hat{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.
\]  

(a) Show that it *does not* satisfy the normal property.

(b) Show that it *can not* be diagonalized.

*Hint:* Compute the set of all eigenvectors, and explain why it does not form a complete basis for the vector space.

3. **Bloch sphere.**– Show that the arbitrary single-qubit state

\[
|\Psi\rangle = \cos \left( \frac{\theta}{2} \right) |0\rangle + e^{i\phi} \sin \left( \frac{\theta}{2} \right) |1\rangle
\]  

is represented by the unit vector \( \hat{n} \) in spherical coordinates

\[
\hat{n} = \cos (\phi) \sin (\theta) \hat{x} + \sin (\phi) \sin (\theta) \hat{y} + \cos (\theta) \hat{z}
\]  

when plotted in the Bloch sphere.

4. **Active rotation.**– Consider the operator

\[
\hat{R}_z(\gamma) = e^{-i\frac{\gamma}{2} \hat{Z}}.
\]  

(a) Write it in explicit matrix form.

(b) Compute

\[
|\Psi'\rangle = \hat{R}_z(\gamma) \cdot |\Psi\rangle
\]  

where the state \( |\Psi\rangle \) is given by Eq. (2). Sketch the states \( |\Psi\rangle \) and \( |\Psi'\rangle \) in the Bloch sphere and give an interpretation for what \( \hat{R}_z(\gamma) \) acts.