Introduction to Quantum Computing

Assignment 4 - Due March 12

Algorithms that exploit quantum parallelism (Deutch, Bernstein-Vazirani, Simon)

1. **Deutch's algorithm for a constant function.**– Describe Deutch's algorithm for the specific case $f(x) = 1$, with $x = 0, 1$. Write down all the kets $|\psi_i\rangle$ explicitly after each gate is applied and explain how Deutch can reveal that $f(x)$ is constant in one run.

2. **Hadamard gate applied to many qubits.**– Use the general formula

$$H^{\otimes n} |x\rangle_n = \frac{1}{\sqrt{2^n}} \sum_z (-1)^{x \cdot z} |z\rangle_n \tag{1}$$

   to calculate:

   (a) $H^{\otimes 3} |010\rangle$ in terms of computational basis states.

   (b) Expand the following expression:

$$H^{\otimes n} \left(\frac{|x\rangle_n + |\bar{x}\rangle_n}{\sqrt{2}} \right), \tag{2}$$

   where $x = x_{n-1} x_{n-2} \cdots x_0$ and $\bar{x} = \bar{x}_{n-1} \bar{x}_{n-2} \cdots \bar{x}_0$, with $\bar{x}_i = x_1 \oplus 1$.

3. **Bernstein-Vazirani for 6 bits.**– Write down the complete circuit (including oracle U_f in terms of CNOT operations) that executes Bernstein-Vazirani for $f(x) = a \cdot x$ with $a = 100101$.

4. **Simon's algorithm for 3 bits.**– Set up Simon's algorithm for $f : \{0, 1\}^3 \to \{0, 1\}^2$ with $a = 011$. Describe a scenario of measurement outcomes that allow determining "a" with only two runs. For each run, write explicitly the states $|\psi_3\rangle$ (just before the last Hadamard application) and $|\psi_4\rangle$ (right before readout of the input register).

5. **Probabilities for success in Simon's.**– For problem 4 above, calculate:

   (a) The probability of finding "a" in 2 runs. Compare to the classical probability of finding "a" in 2 runs.

   (b) The probability of finding "a" in 3 runs. Compare to the classical probability for success in 3 runs.