Survey of quantum physics

Stern-Gerlach experiment, electron spin and analogy with polarization of light

Stern and Gerlach, 1922

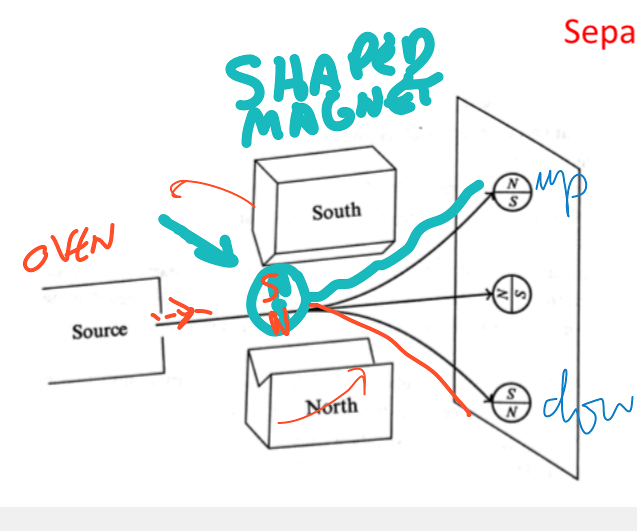
O. Stern and W. Gerlach, Zeitschrift für Physik **8**, 110 (1922)

- A beam of silver atoms deflected by a magnetic field gradient. What do we expect from classical physics?

Ag = (Kr)$(5d)^{10}(4s)^1$

Stern-Gerlach experiment

What is the result of this experiment?
Separate into groups of 5 and discuss.



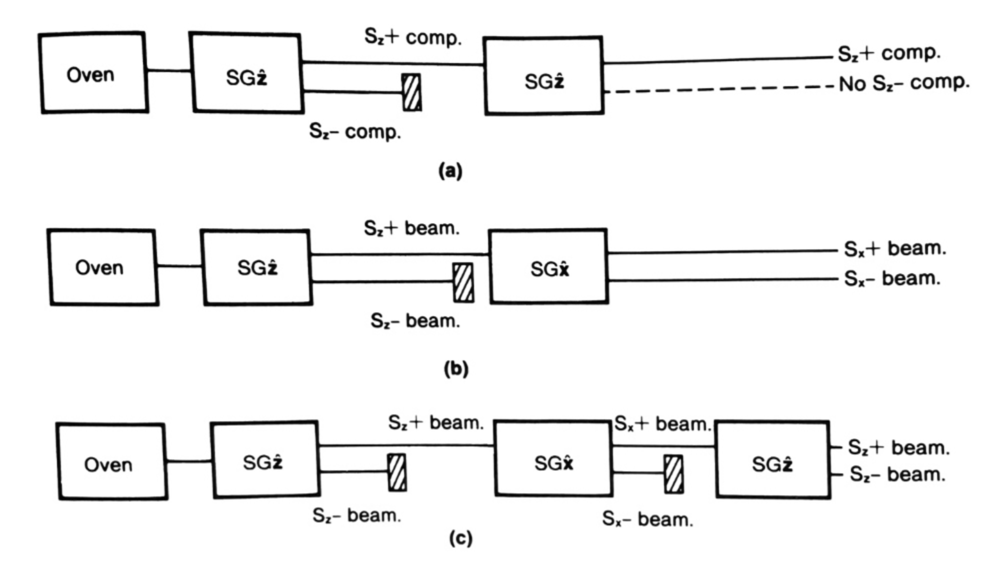
Hints:

- The Ag atoms come out of the oven with random magnetic orientation.
- The force that the shaped magnet induces on each atom is proportional to the projection of the atom's magnetic moment along the vertical.

What we expect from classical physics / What is actually observed

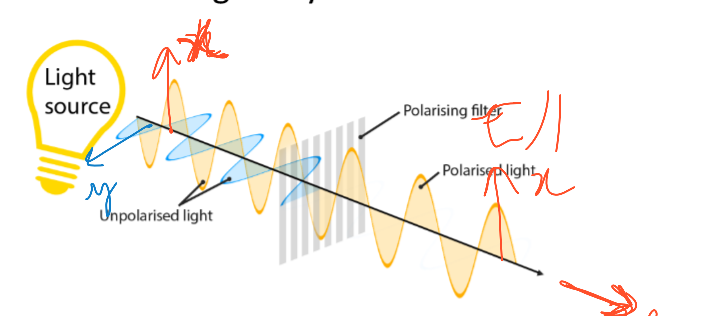
Possible explanation: The oven creates Ag atoms with magnetic moment either up or down. To check this, Stern and Gerlach rotated the shaped magnet, expecting to get a single peak. But they got the same two peaks, split along the new orientation of the shaped magnets!

Sequential Stern-Gerlach experiments



Explanation of S-G: Just like polarization of light!

- Light is an electromagnetic wave: If it propagates along z, its electric field can can be oriented along x or y:

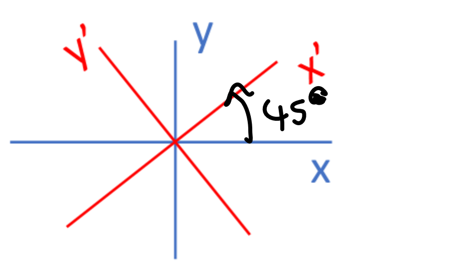


$$\boldsymbol{E} = E_0 \hat{\boldsymbol{x}} \cos(kz - \omega t) \qquad \text{“x-polarized”}$$

$$\boldsymbol{E} = E_0 \hat{\boldsymbol{y}} \cos(kz - \omega t) \qquad \text{“y-polarized”}$$

Explanation of S-G: Analogy with polarization of light

$$S_z + \ \text{atoms} \longleftrightarrow \text{x-polarized light}$$
$$S_z - \ \text{atoms} \longleftrightarrow \text{y-polarized light}$$



$$S_x + \ \text{atoms} \longleftrightarrow \text{x'-polarized light}$$
$$S_x - \ \text{atoms} \longleftrightarrow \text{y'-polarized light}$$

Explanation of S-G: Analogy with polarization of light

- But, for light we know that:

$$E_0 \hat{\boldsymbol{x}}' \cos(kz - \omega t) = E_0 \left[\frac{1}{\sqrt{2}} \hat{\boldsymbol{x}} \cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{\boldsymbol{y}} \cos(kz - \omega t) \right]$$

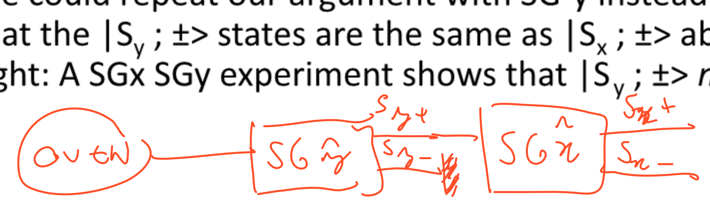
$$E_0 \hat{\boldsymbol{y}}' \cos(kz - \omega t) = E_0 \left[-\frac{1}{\sqrt{2}} \hat{\boldsymbol{x}} \cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{\boldsymbol{y}} \cos(kz - \omega t) \right]$$

- This suggests the following *conjecture*:

$$| S_x; + \rangle = + \frac{1}{\sqrt{2}} | S_z; + \rangle + \frac{1}{\sqrt{2}} | S_z; - \rangle$$
$$| S_x; - \rangle = - \frac{1}{\sqrt{2}} | S_z; + \rangle + \frac{1}{\sqrt{2}} | S_z; - \rangle$$

But something is missing: What about the states created by SG-y?

- We could repeat our argument with SG-y instead of SG-x, and conclude that the $|S_y ; \pm\rangle$ states are the same as $|S_x ; \pm\rangle$ above. But that can not be right: A SGx SGy experiment shows that $|S_y ; \pm\rangle$ *must* be different!



- Analogy with light rescues us again here: Consider *right-circularly* polarized light:

$$\boldsymbol{E} = E_0 \left[\frac{1}{\sqrt{2}} \hat{\boldsymbol{x}} \cos(kz - \omega t) - \frac{1}{\sqrt{2}} \hat{\boldsymbol{y}} \sin(kz - \omega t) \right]$$
$$= E_0 \text{Re} \left[\frac{1}{\sqrt{2}} \hat{\boldsymbol{x}} e^{i(kz - \omega t)} + \frac{i}{\sqrt{2}} \hat{\boldsymbol{y}} e^{i(kz - \omega t)} \right]$$

States SGy: Analogous to circularly polarized light!

$$S_y + \ \text{atom} \longleftrightarrow \text{right-circularly-polarized light}$$
$$S_y - \ \text{atom} \longleftrightarrow \text{left-circularly-polarized light}$$

$$| S_y; + \rangle = + \frac{1}{\sqrt{2}} | S_z; + \rangle + \frac{i}{\sqrt{2}} | S_z; - \rangle$$
$$| S_y; - \rangle = + \frac{1}{\sqrt{2}} | S_z; + \rangle - \frac{i}{\sqrt{2}} | S_z; - \rangle$$

Summary

- Quantum mechanical states are to be represented by a vector (a "ket") in an abstract complex vector space (same one you learned in 1st year calculus/linear algebra, but now the coefficients can be complex!).
- In the next few classes we will develop the tools necessary to describe quantum states: complex numbers, linear algebra, and Dirac's notation (AKA bra-ket notation).

|KET⟩

⟨BRA|

N qubits $N = 10^6$

IN a QC: $2^N = 2^{10^6}$ HUGE!

READ-OUT, N outcomes