Introduction to Quantum Computing

Assignment 5 - Due March 27th

Quantum algorithms with IBM-Q

Instructions:

- All problems below should be solved with Qiskit. Once you're done, submit your python notebooks by clicking here.

1. **Quantum cryptography.**– Write a Qiskit code that implements BB84 with 15 qubits. You will play Alice.

   (a) First use the local simulator *qasm_simulator* to play Bob. State clearly the value of the one-time pad that you got.

   (b) Second, use the real device *ibmq_16_melbourne* to play Bob. Again, clearly state the one-time pad that you got.

2. **Quantum cloning.**

   (a) Design a quantum circuit that clones the states $|+\rangle$ and $|-\rangle$. I.e., design a unitary U that achieves the following:

   $$U(|+\rangle|0\rangle) = |+\rangle|+\rangle, \quad U(|-\rangle|0\rangle) = |-\rangle|-\rangle. \tag{1}$$

   (b) Apply this U to $|0\rangle$ and $|1\rangle$. Evaluate the "cloning fidelity" by computing

   $$F_\psi = |\langle\psi|\langle\psi|U(|\psi\rangle|0\rangle)|^2, \tag{2}$$

   with the *qasm_simulator* , for $|\psi\rangle = |0\rangle$ and $|\psi\rangle = |1\rangle$. How good is the cloning?

3. **One-qubit tomography.**– Write the three circuits necessary to measure the projections of the state

   $$|\psi\rangle = \cos\left(\frac{\pi}{8}\right)|0\rangle + \sin\left(\frac{\pi}{8}\right)|1\rangle, \tag{3}$$

   along the x, y, and z axes of the Bloch sphere. Do this for the simulator and for a real device using 1000 shots. Use the function *plot_bloch_vector([px,py,pz])* to display the Bloch vectors. How do they compare?

4. **Error correction with the three-qubit bit-flip code.**– The circuit shown below is the "automated" version of the three-qubit bit-flip error correction code described in class. The last five gates – Two CNOTS and three Toffolis – perform the same "correction protocol" achieved by the conditional measurement operations shown in class. The final state $|\Phi\rangle$ of the ancillas remains $|00\rangle$ if no error happens, and becomes $|10\rangle$, $|11\rangle$, $|01\rangle$ when one of the qubits 2, 1, 0 flips, respectively. If another round of error correction is needed the ancilla state $|\Phi\rangle$ must be reset to $|00\rangle$ (currently not allowed in real IBM-Q devices).

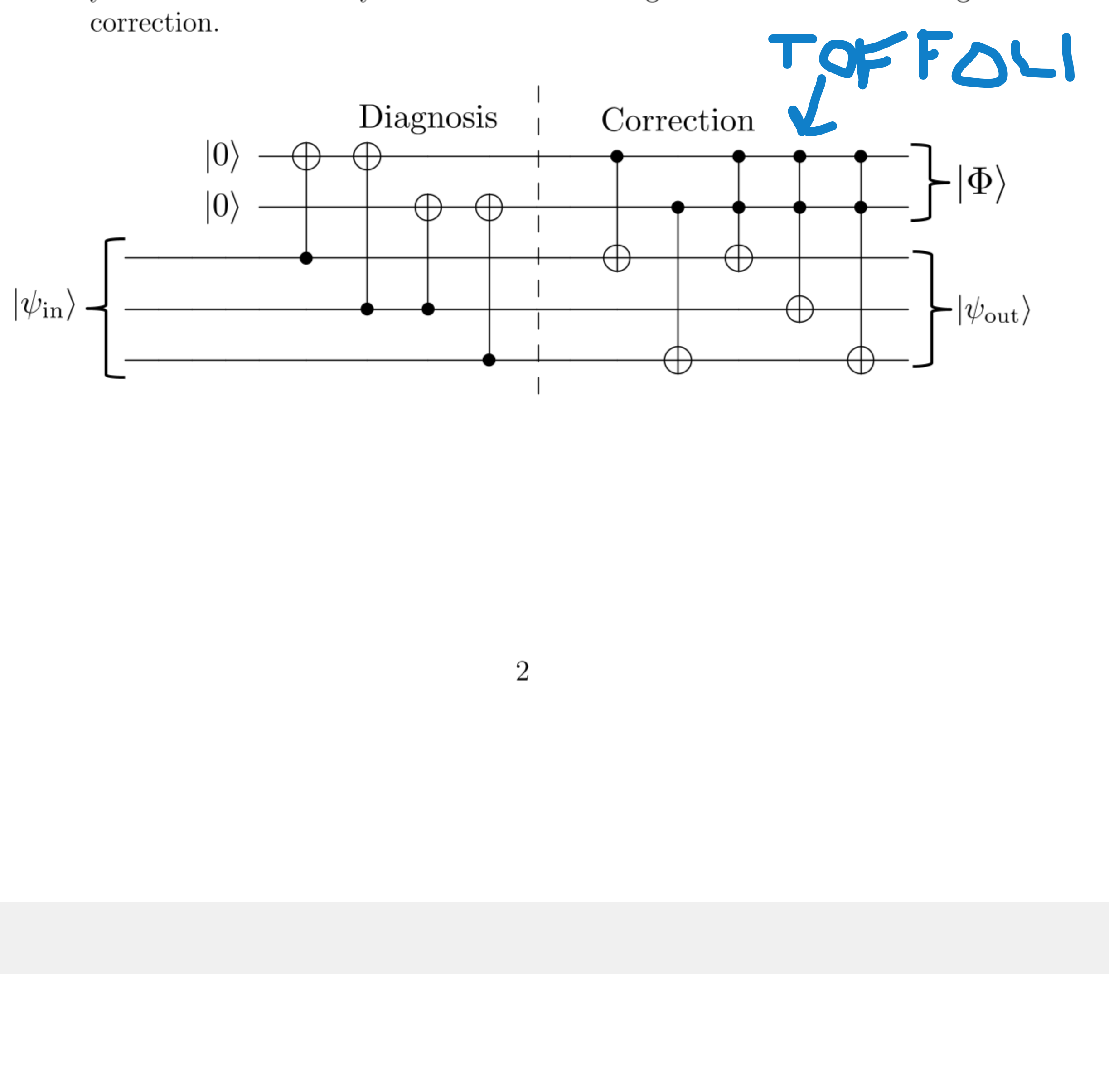
   (a) Implement this algorithm for

   $$|\psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{4}$$

   in the simulator and check that it works by intentionally flipping each one of the three qubits in $|\psi_{\text{in}}\rangle$ before the error correction procedure. Measure $|\psi_{\text{out}}\rangle$ in the computational basis and check that you get close to 50% probability for 000 and 111.

   (b) Run the same circuit in a real device (no need to add the intentional flips!), and measure $|\psi_{\text{out}}\rangle$. How good is it?

   (c) As a comparison, run the same algorithm in the same real device *without* the last five "correction protocol" gates. How does the computational basis probabilities for $|\psi_{\text{out}}\rangle$ compare to the ones found in item (b)? Conclude by saying whether you think the device you used has low enough noise to take advantage of error correction.



5. **Three-bit phase estimation**.

   (a) Implement the circuit below for $U = R_y(\theta)$ with $\frac{\theta}{4\pi}$ exactly represented by three base-2 decimals, $\frac{\theta}{4\pi} = 0.j_1j_2j_3$ (your choice of $j_1j_2j_3$). Note that $\frac{\theta}{4\pi}$ is the associated phase ϕ_u for $|u\rangle = |-i\rangle$, since

   $$R_y(\theta)|-i\rangle = e^{+i\frac{\theta}{2}}|-i\rangle = e^{+2\pi i\frac{\theta}{4\pi}}|-i\rangle. \tag{5}$$

   Input $|\psi\rangle = |-i\rangle$, and run the circuit in the simulator and a real device to compare.

   (b) Run the circuit again for $\frac{\theta}{4\pi} = 0.j_1j_2j_3 - \epsilon$, with ϵ small. If you run the circuit many times does it return the $0.j_1j_2j_3$ closest to the correct answer $\frac{\theta}{4\pi}$? How does the real device compare to the simulator?