Adiabatic quantum computing

Alternative to the standard “gate model” of quantum computing

The Hamiltonian operator: Represents energy in quantum theory

- We learned that Hamiltonian operators describe “dynamical” things that we can measure. ENERGY is described by an operator called “Hamiltonian”, denoted $\hat{H}$ (in a rigorous, for $\hat{H}$ = to be confused with Hamilton).

Example: Single spin, like an “artificial atom”

$$\hat{H} = -\frac{1}{2} \gamma I \otimes \sigma_z$$

Two qubits, Artificial molecule

$$\hat{H} = -\frac{1}{2} \gamma I \otimes \sigma_z + \frac{1}{2} \gamma I \otimes \sigma_z$$

Eigenstates of the Hamiltonian: Ground and excited states

- The system evolves in time $t$ along the “fast” defined energy $E_i(t)$, with the associated eigenvector. The eigenvector associated to the lowest eigenvalue is called ground state, denoted $|\psi_i\rangle$. These states are special because they are quite stable. They only change if energy is applied to them.

$$H |\psi_i\rangle = E_i |\psi_i\rangle$$

- The other states with higher energy are called excited states.

Time evolution of a quantum state

- The time evolution of a quantum state is described by Schrödinger’s eqn (the equivalent of Navier-Stokes for quantum mechanics):

$$\frac{d}{dt} |\psi(t)\rangle = -i \frac{\hat{H}}{\hbar} |\psi(t)\rangle$$

- When $H$ is independent of time, this eqn is quite easy to solve. The general solution is:

$$|\psi(t)\rangle = e^{-i \hat{H} t/\hbar} |\psi(0)\rangle$$

Time evolution leads to interference and tunnelling

If $|\psi(t)\rangle$ is an eigenstate of $\hat{H}$, $|\psi(t)\rangle$ only gets a global phase:

$$e^{-i \hat{H} t/\hbar} |\psi(0)\rangle = e^{-i E \hat{H} t/\hbar} |\psi(0)\rangle$$

Consider what happens when $|\psi(t)\rangle$ is not an eigenstate of $\hat{H}$:

$$|\psi(t)\rangle = e^{-i \hat{H} t/\hbar} |\psi(0)\rangle$$

Example:

$$\hat{H} = -\frac{1}{2} \gamma I \otimes \sigma_z$$

$$|\psi(t)\rangle = e^{-i \hat{H} t/\hbar} |\psi(0)\rangle$$

Note the global phases in the coeffs:

$$|\psi(t)\rangle = e^{-i \hat{H} t/\hbar} |\psi(0)\rangle = e^{-i E \hat{H} t/\hbar} |\psi(0)\rangle = e^{-i E \hat{H} t/\hbar} |\psi(0)\rangle$$

$$\Psi(0) = \begin{pmatrix} a_0 & b_0 \end{pmatrix}$$

$$\Psi(t) = \begin{pmatrix} a_t & b_t \end{pmatrix}$$

- With slow time dependence

But what if the changes in time $\Delta \hat{H}$ of Schrödinger’s eqn is much more difficult. But as times $t$ get larger, these equations can be solved numerically:

$$\frac{d}{dt} |\psi(t)\rangle = -i \frac{\hat{H}}{\hbar} |\psi(t)\rangle$$

Assume $|\psi(t)\rangle$ changes only linearly with $t$ and the energy levels $E_i$ do not change appreciably with this delay. If $T$ is the period in one of the eigenvector $|\psi(t)\rangle = e^{-i \hat{H} t/\hbar}$, they remain in the “adiabatic regime”:

$$T \ll \frac{1}{\Delta E}$$

Adiabatic theorem

The eigenstates are the instantaneous eigenstates $|\psi(t)\rangle$ of $\hat{H}(t)$, such that at $t = 0$, $|\psi(0)\rangle$ and $|\psi(t)\rangle$ are identical.

$$\int |\psi(t)\rangle \langle \psi(t) | \hat{H}(t) |\psi(t)\rangle \langle \psi(t) | dt = \langle \hat{H}(t) | \psi(t) \rangle$$

for $t \geq 0$.

Adiabatic quantum computation (AQC)

- Design a Hamiltonian that changes in time so that $|\psi(t)\rangle$ has a known final state as its ground state, at $t = T$.

- And for $|\psi(t)\rangle$ has a ground state that includes the solution to the problem you want to solve. I.e., for adiabatic quantum solvers

$$\Delta E \ll \frac{E_i}{T}$$

- So that performing a measurement (at a particular time) will give the desired answer to the problem.

$$|\psi(T)\rangle = \sum_i c_i |\phi_i\rangle$$

$$\hat{H} |\psi(T)\rangle = \sum_i c_i \hat{H}_i |\phi_i\rangle$$

Example: Adiabatic version of Deutsch-Jozsa algorithm

- Recall gate model Deutsch-Jozsa

- Adiabatic version: