

# Phys 507A - Solid State Physics I

## Assignment 5: Interaction between quasiparticles. Due Apr. 6th

1. *Deformation potential in a triply degenerate (p-like) band.*

The application of stress to a crystal can split a set of degenerate bands. Crystal symmetry dictates how each band is coupled to the other bands, i.e., how the deformation interaction depends on the elements of the strain tensor. In a cubic crystal, a triply degenerate band with p-symmetry has the following deformation potential Hamiltonian, known as the Pikus-Bir Hamiltonian:

$$\mathcal{H}_{\text{PB}} = a\text{Tr}(\boldsymbol{\epsilon})\mathbf{1}_{3\times 3} + \begin{pmatrix} b(\epsilon_{xx} + \epsilon_{yy} - 2\epsilon_{zz}) & \sqrt{\frac{3}{2}}d(\epsilon_{xz} + i\epsilon_{yz}) & \sqrt{\frac{3}{2}}d(-\epsilon_{xz} + i\epsilon_{yz}) \\ \sqrt{\frac{3}{2}}d(\epsilon_{xz} - i\epsilon_{yz}) & -\frac{1}{2}b(\epsilon_{xx} + \epsilon_{yy} - 2\epsilon_{zz}) & \frac{3}{2}b(\epsilon_{xx} - \epsilon_{yy}) - i\sqrt{3}d\epsilon_{xy} \\ \sqrt{\frac{3}{2}}d(-\epsilon_{xz} - i\epsilon_{yz}) & \frac{3}{2}b(\epsilon_{xx} - \epsilon_{yy}) + i\sqrt{3}d\epsilon_{xy} & -\frac{1}{2}b(\epsilon_{xx} + \epsilon_{yy} - 2\epsilon_{zz}) \end{pmatrix},$$

operating on the basis of the three degenerate “p-like” ( $L = 1$ ) and  $k = 0$  band states  $|0\rangle = |z\rangle$ ,  $|1\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle)$ , and  $|-1\rangle = \frac{1}{\sqrt{2}}(|x\rangle - i|y\rangle)$ . The constant  $a$  is a hydrostatic deformation potential, while  $b$  and  $d$  are shear deformation potentials. Note that this becomes an electron-phonon interaction when we plug the strain as a function of the phonon creation and annihilation operators.

- (a) Assume uniaxial strain along one of the [001] directions, i.e.  $\epsilon_{\alpha\alpha} = \epsilon$  for only one  $\alpha$ , all other strains equal to zero. Find the eigenenergies of the Hamiltonian, and show that the hydrostatic energy shift is  $a\epsilon$ , while the energy splitting of the states is equal to  $|3b\epsilon|$ .
- (b) For uniaxial strain along the [111] direction given by  $\epsilon'$ , show that the splitting of the states is equal to  $|\sqrt{3}d\epsilon'|$ .
- (c) Show that this Hamiltonian gives a nonzero electron-phonon interaction for longitudinal phonons – phonons with polarization along  $\mathbf{k}$ . Show that the shear term also gives a non-zero contribution for transverse phonons.

*Hint: Uniaxial strain along the [111] direction is given by*

$$\boldsymbol{\epsilon} = \epsilon' \begin{pmatrix} \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}} \\ \sqrt{3} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}} \\ \sqrt{3} \end{pmatrix}. \quad (1)$$

2. *Temperature dependence of the electron-phonon scattering time in a metal at low temperature.*

The electron-phonon scattering process has to obey the following conservation rules:

$$\mathbf{q} = \mathbf{q}' \pm \mathbf{k} \quad (2)$$

$$E_q = E_{q'} \pm \hbar\omega_k. \quad (3)$$

Moreover, at low temperatures (much smaller than the Debye temperature), the physically relevant phonons for e-ph scattering have wavevectors  $k$  of the order of  $k_B T / (\hbar v)$ . This is because the phonons that are available for absorption are the thermally excited ones; and the phonons that are generated by spontaneous emission can not have energy much larger than  $k_B T$  (due to Pauli exclusion electrons can only give away energy of the order of  $k_B T$ , the smearing of the Fermi sea).

- (a) Use this information to draw a diagram of electron-phonon scattering in a spherical Fermi surface, i.e., for a given electron  $\mathbf{q}$ , draw the region where  $\mathbf{q}'$  can be located. Conclude that the sum of these  $\mathbf{q}'$ 's, “the phase space for scattering” has to scale as  $T^2$ .
- (b) How does the rate for e-ph scattering due to the deformation potential interaction scales with temperature?
- (c) How does the rate for e-ph scattering due to the piezo-electric interaction scales with temperature?

3. *Length scale for the absorption of light in a semiconductor.*

The rate for interband transitions induced by photons in a semiconductor was derived to be

$$\frac{1}{\tau} = \sqrt{\hbar\omega - E_{\text{gap}}} \frac{e^2 m_r^{3/2}}{\pi \hbar^2 \epsilon m_e^2} \frac{|\langle v | \mathbf{p} | c \rangle \cdot \hat{\boldsymbol{\eta}}|^2}{\hbar\omega}. \quad (4)$$

- (a) Convert this to an absorption length. That is, show that for a constant fluence of photons hitting a surface at  $x = 0$ , the probability of a photon not being absorbed at distance  $x$  from the surface is given by

$$P(x) = e^{-x/l}, \quad (5)$$

where  $l$  is the absorption length. Write down an expression for  $l$  using Eq. (4).

*Hint: Once inside the material, the photon moves with velocity  $c/n$ .*

- (b) Estimate the absorption length for the case  $E_{\text{gap}} = 2$  eV,  $\epsilon/\epsilon_0 = 10$ , and effective masses all of the order of the free electron's mass  $m_e$ . Assume the photon frequency is given by  $\hbar\omega/E_{\text{gap}} = 1.1$ . The matrix element for a semiconductor is

usually given in terms of the unitless oscillator strength:

$$f = \frac{2|\langle v|\mathbf{p}|c\rangle|^2}{mE_{\text{gap}}}, \quad (6)$$

which is of the order of unity for typical semiconductors.

4. *Grüneisen parameters in one dimension.*

Consider a one-dimensional array of  $N$  atoms interacting through pair potentials  $\phi(r)$ . In this problem you will include interaction between first and second nearest neighbor atoms.

- (a) Compute the frequency of normal modes of vibration  $\omega_k$  as a function of  $\phi''(a)$  and  $\phi''(2a)$ , where  $a$  is the equilibrium interatomic distance.
- (b) Assume that the array is subject to hydrostatic strain  $\varepsilon$ , i.e., its equilibrium interatomic distance changes to  $(1 + \varepsilon)a$ . Compute the  $k$ -dependent Grüneisen parameter, defined by

$$\gamma_k = \frac{1}{\omega_k} \left. \frac{\partial \omega_k}{\partial \varepsilon} \right|_{\varepsilon=0}. \quad (7)$$

Show that  $\gamma_k$  is actually *independent of  $k$*  when only nearest-neighbor interactions are present.

*Hint: Don't make the long wavelength approximation here (i.e., don't drop terms of order  $k^2$ ). Label your normal modes by  $k(1 + \varepsilon)a \rightarrow k$ , i.e., the phase difference from one atom to the other. Equation (7) is the volume dependence of the same mode (same interatomic phase difference). If  $k$  depends on  $\varepsilon$ , we will get an additional (incorrect) contribution to  $\gamma_k$ .*