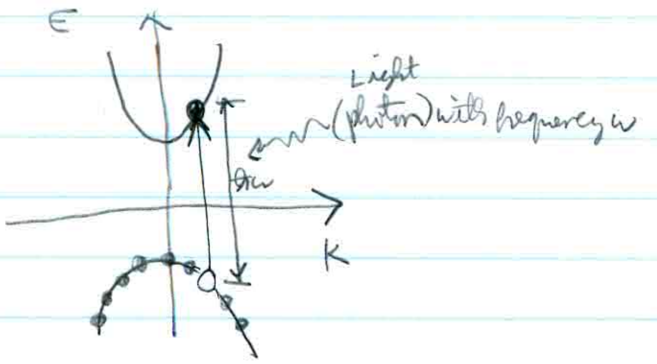
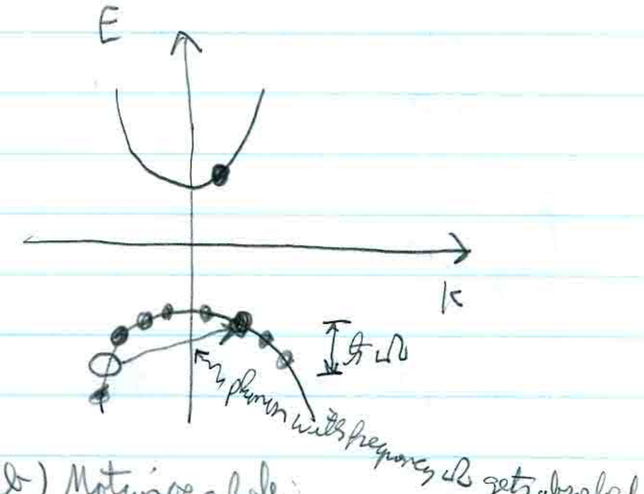
Lecture 7

Quasiparticles: Electrons, holes, and excitons



(a) A photon is absorbed by a semiconductor and an electron-hole pair is created



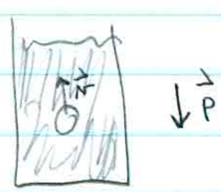
(b) Motion of a hole

An electron may scatter into this hole state, leaving a hole at its original state. This is equivalent to a hole moving with opposite $\vec{K}$.

A "hole" is as good a particle as a real electron. The only distinction is:

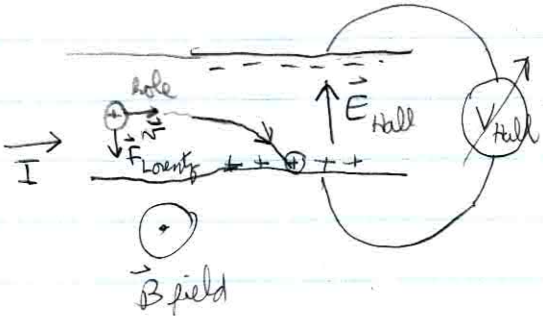
	Electron	hole
Charge	$-\lvert e\rvert$	$+\lvert e\rvert$
Crystal momentum	$+\hbar\vec{K}$	$-\hbar\vec{K}$
Mass	m_c^*	m_v^*

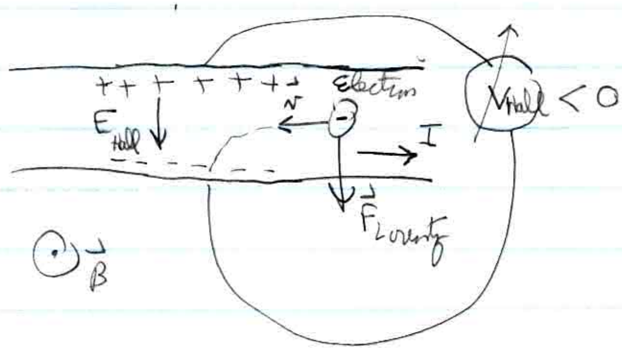
A good analogy is a bubble moving in a liquid



It is much more convenient ~~to~~ to think of the bubble going up rather than thinking of the liquid "going down".

How to determine the sign of the charge carriers? Hall voltage!

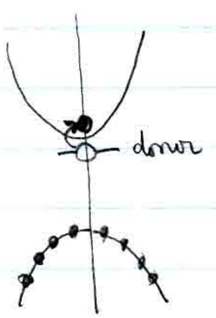




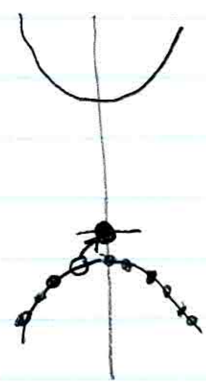
Lorentz Force $\vec{F}_{Lorentz} = \frac{q}{c} \vec{v} \times \vec{B}$

The sign of the Hall Voltage depends on the sign of the charge carriers!

How to get charge carriers in a semiconductor?



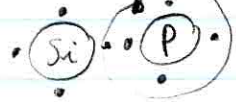
I "Binding energy" Energy necessary to break the "gigantic H-atom".



Dope with a Donor: Impurity atom has larger valence than lattice atom.

Extra electron! Looks like a gigantic 1s state for an H atom

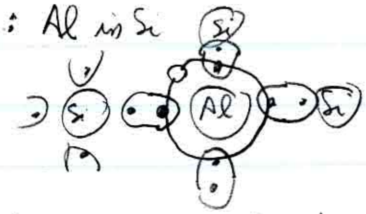
Eg. P in Si:



Negative charge carriers!

Dope with acceptor: Impurity has smaller valence than lattice atoms

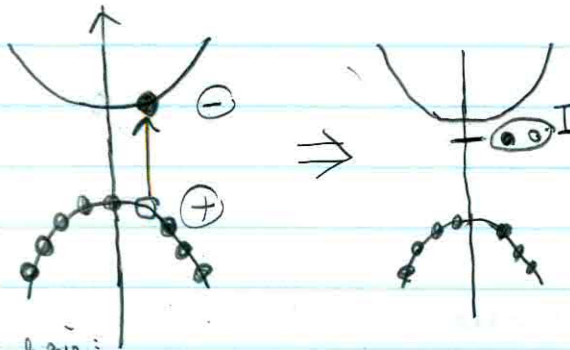
Eg: Al in Si



Positive charge carriers!

Excitons

Wannier's exciton: (For delocalized electrons and holes).



Exciton binding energy. The e-h pair now can move together as (• o).

Electron-hole pair: Two free carriers, one ⊖, other ⊕.

They attract each other and bind forming a "large H-atom": Exciton.

Exciton: A atom formed by an electron orbiting around a hole:

$\vec{r} = \vec{r}_e - \vec{r}_h$

$\vec{R} = \frac{\vec{r}_e + \vec{r}_h}{2}$

$$H = \frac{\vec{P}^2}{2(m_v^* + m_c^*)} + \frac{\vec{p}^2}{2m_{reduced}} - \frac{e^2}{\varepsilon}\frac{1}{|\vec{r}|}$$

with $m_{reduced} = \frac{m_c^* m_v^*}{m_c^* + m_v^*}$.

Try $\psi_{\substack{Singlet\\ n=1}}(\vec{R}, \vec{r}) = \frac{e^{i\vec{K}\cdot\vec{R}}}{\sqrt{V}} \frac{e^{-\frac{r}{a^*}}}{\sqrt{\pi a^{*3}}} \left(\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}\right)$ (Exciton ground state!)

Excited states for $n > 1$; Generally: (ψ_n can be Triplet or singlet for $n > 1$, but for $n = 1$ can only be singlet!).

$$H\psi_n = \left[\frac{\hbar^2 K^2}{2(m_v^* + m_c^*)} - \frac{Ry'}{n^2}\right]\psi_n$$

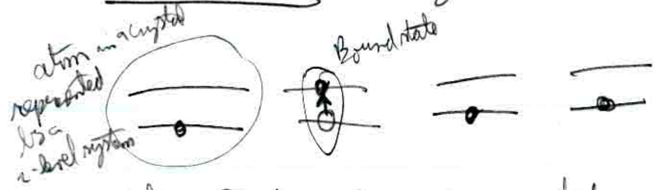
$$Ry' = \frac{1}{2}\frac{e^2}{\varepsilon a^*}, \quad a^* = \frac{\varepsilon \hbar^2}{e^2 m_{reduced}}$$

(Just make $e^2 \to \frac{e^2}{\varepsilon}$ in H atom formulas!)

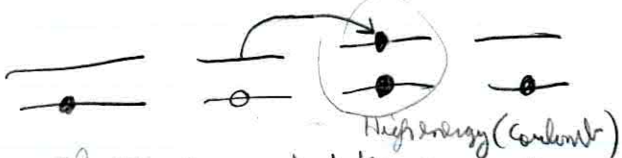
	ε	Exciton binding energy	a^*
Si	11.4	12 meV	50 Å
GaAs	13.1	4 meV	150 Å

Frenkel's exciton (Localized electron and holes)

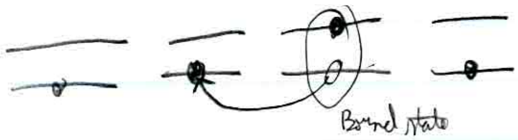
Exciton is so tightly bound that it sits in the same lattice site.



Localized Electron hole pair is excited



Electron hops, but the energy of one of the atoms is now too high



To relieve this Coulomb energy, electron moves to nearest neighbour.

$\Rightarrow$ As a result, exciton can move across crystal.

The concept of a quasiparticle

The fundamental electron Hamiltonian: (assuming lattice atoms are frozen)

$$H = \sum_i \left[\frac{\vec{p}_i^{\,2}}{2m} + U(\vec{r}_i) \right] + \underbrace{\sum_{i<j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|}}_{\text{Quite strong! Why can I neglect?}}$$

The quasiparticle concept: I can ignore the Coulomb repulsion between electrons provided I think of electrons and holes not as single stationary states, but rather as a wavepacket of Bloch states.

$\Rightarrow$ Not an eigenstate, finite lifetime τ.

$\boxed{\text{e–e interaction}}$ ① may affect m_c^*, m_v^* and gives $\varepsilon > 1$ (screening).

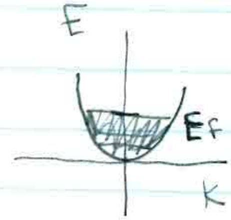
② electron and hole excitations get a "lifetime" τ.

Horse is a particle $\Rightarrow$ (Horse + dust around it) is a quasiparticle.

Isotropic Fermi gas at $T=0$

$$\beta \equiv k_B T$$

$$f(E) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}, \quad \mu(T=0) \equiv \varepsilon_F$$



When $T=0 \Rightarrow f(\varepsilon) = \Theta(\varepsilon_F - \varepsilon)$.

$$N = \int_0^\infty dE\, D(E) f(E) = \int_0^{E_F} dE\, D(E)$$

$$D(E)\, dE = \frac{2 \cdot 4\pi k^2}{\frac{(2\pi)^3}{V}} dk = \frac{V}{\pi^2} \frac{k^2}{\left|\frac{dE}{dk}\right|} dE = \frac{V}{\pi^2} \frac{k^{\cancel{2}}}{\frac{\hbar^2}{m} \cancel{k}} dE = \underbrace{\left(\frac{V}{\pi^2} \frac{m}{\hbar^2} \sqrt{\frac{2mE}{\hbar^2}}\right)}_{D(E)} dE$$

$$N = \frac{V}{\pi^2} \frac{m}{\hbar^2} \sqrt{\frac{2m}{\hbar^2}} \int_0^{E_F} dE \sqrt{E} = \frac{V}{\pi^2} \sqrt{2}\, \frac{m^{3/2}}{\hbar^3} \frac{2}{3} E_F^{3/2} \Rightarrow \left(\frac{N}{V}\right) = \frac{2^{3/2}}{3\pi^2} \frac{m^{3/2}}{\hbar^3} E_F^{3/2}$$

$$E_F = \left(\frac{3\pi^2}{2^{3/2}} \frac{\hbar^3}{m^{3/2}}\right)^{2/3} \left(\frac{N}{V}\right)^{2/3} = \frac{\hbar^2}{2m} \underbrace{(3\pi^2)^{2/3} n^{2/3}}_{k_F^2} \quad // \quad \left(n \equiv \frac{N}{V} \text{ is density}\right)$$

$$E_F = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \propto n^{2/3} \; //$$

$n \sim 10^{23}\, cm^{-3} \Rightarrow E_F \sim 10\, eV$. Also, $E_F = k_B T_F \Rightarrow T_F \sim 10^4 - 10^5\, K$!

introduce at 4

Metals are "very quantum" even at Room Temp.

Pressure and Compressibility of the Fermi gas

$$P = -\frac{\partial U}{\partial V} \; ; \quad U = \int_0^{E_F} dE\, E\, D(E) = \frac{V}{\pi^2} \sqrt{2}\, \frac{m^{3/2}}{\hbar^3} \int_0^{E_F} dE\, E^{3/2} = \underbrace{\frac{V\sqrt{2}}{\pi^2} \frac{m^{3/2}}{\hbar^3} \frac{2}{5} E_F^{5/2}}_{\frac{3}{5}\frac{N}{E_F^{3/2}}} = N \frac{3}{5} E_F$$

$$U = \frac{V\sqrt{2}}{3\pi^2} \frac{m^{3/2}}{\hbar^3} \frac{2}{5} \left(\frac{3\pi^2}{2^{3/2}} \frac{\hbar^3}{m^{3/2}}\right)^{5/3} \frac{N^{5/3}}{V^{5/3}} V = \frac{3}{5} \left(\frac{3\pi^2}{2^{3/2}} \frac{\hbar^3}{m^{3/2}}\right)^{2/3} \frac{N^{5/3}}{V^{2/3}} = \frac{A}{V^{2/3}}.$$

$$P = -\frac{\partial U}{\partial V} = \frac{2}{3}\frac{U}{V} = \frac{2}{3}\frac{3}{5}\frac{N\epsilon_F}{V} = \frac{2}{5} n \epsilon_F$$ (Quite High Pressure!)

Bulk compressibility: $$B = -V\frac{\partial P}{\partial V} = \frac{5}{3}P = \frac{5}{3}\frac{2}{5} n\epsilon_F = \frac{2}{3} n \epsilon_F$$

Pressure needed to change volume by 1%: $\Delta P = B\frac{\Delta V}{V} = 10^{-2}B = 10^{-2}\times\frac{2}{3}\times 10^{23}\,cm^{-3}\times 10\,eV$
$\sim 10^4$ atm!

Fermi gas of a metal is hard to compress!

why metal doesn't expand indefinitely?

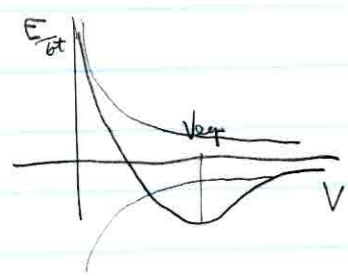
Consider e.g. electrostatic energy: $$E_{attraction} = \int d^3r \underbrace{\left(-\frac{eN}{V}\right)}_{\text{Electron}} \sum_{i=1}^{N_a} \frac{\nu e}{|\vec{r}-\vec{R}_i|}$$

$(\underset{\#\text{ of atoms}}{N_a}\ \underset{\text{valence}}{\nu} = N)$

$$= -\frac{(Ne)^2}{V}\int_0^R \frac{4\pi r^2 dr}{r} = -\frac{(Ne)^2}{V} 4\pi \frac{R^2}{2}$$

$$= -\frac{(Ne)^2}{V}\Big(\underbrace{\tfrac{4}{3}\pi R^3}_{V}\Big)^{\frac{2}{3}} \frac{2\pi}{(\frac{4}{3}\pi)^{2/3}} = -\left(\frac{2^{3/2}\pi^{3/2}}{\frac{2^2}{3}\pi}\right)^{2/3}\frac{(Ne)^2}{V^{1/3}}$$

$$E_{attrac} = -\frac{B}{V^{1/3}}$$



$$E_{Total} = \frac{A}{V^{2/3}} - \frac{B}{V^{1/3}}$$

$$\left.\frac{dE_{Total}}{dV}\right|_{V=V_{eq}} = 0 \Rightarrow V_{equilibrium}.$$

$\Rightarrow$ Complete analysis of cohesiveness of metals will be done in P507B.