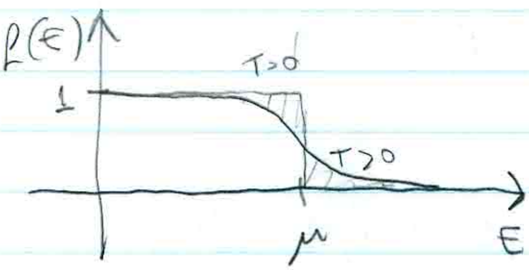
# Lecture #8

## Fermi gas at finite temperature




$$f(E-\mu) = \frac{1}{e^{\beta(E-\mu)}+1} = \left[1 - \frac{e^{\beta(E-\mu)}}{e^{\beta(E-\mu)}+1}\right] = 1 - f(\mu - E)$$

Fraction of "promoted" states equal to fraction of "emptied states".

when $\mu < E_0 << E \Rightarrow f(E) \propto e^{-\beta E}$ (Maxwell-Boltzmann, "classical limit")

If number of particles is fixed:

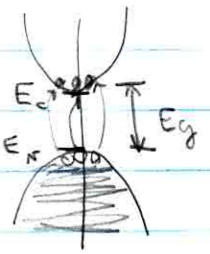
$$N = \langle N \rangle = \int_0^\infty dE\, D(E) \frac{1}{e^{\beta(E-\mu)}+1} \Rightarrow N = N(T,\mu) \Rightarrow \mu = \mu\left(T, \frac{N}{V}\right)$$

very small correction at room T!

$$\boxed{\mu(T,n) = E_F\left[1 - \frac{1}{3}\left(\frac{\pi k_B T}{2E_F}\right)^2 + O(T^4)\right]} \qquad E_F \equiv \mu(T=0) = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

$\mu < E_F$ for $T>0$, why? Note that $D(E) \propto \sqrt{E}$ increases with $E$. More states available above $\mu$ than below $\mu$. Since $f(E)$ promotes the same number of states that it loses below $\mu$, $\mu$ has to decrease to keep $N$ constant.

## Equilibrium of carriers in semiconductors




Pure semiconductor

If $E_g \lesssim 10 k_B T$, some carriers will be thermally excited

$$N_e = \int_{E_c}^{\infty} f(E) D_c(E) dE$$

$$N_h = \int_{-\infty}^{E_v} [1 - f(E)] D_v(E) dE$$

Where is $\mu$ in the semiconductor?

Assume for now that it is somewhere within the gap, $|\mu - E_c|$ and $|\mu - E_v| \gg k_B T$.

We can approximate:

$$N_e \approx \int_{E_c}^{\infty} dE\, e^{-\beta(E-\mu)} D_c(E) = \int_0^{\infty} dE\, e^{-\beta(E+E_c)} e^{\beta\mu} D_c(E+E_c)$$

$E \to E + E_c$

$$N_h \approx \int_{-\infty}^{E_v} dE\, e^{\beta(E-\mu)} D_v(E) dE = \int_0^{\infty} dE\, e^{+\beta(E_v - E)} e^{-\beta\mu} D_v(E_v - E)$$

$E \to E_v - E$

$$N_e N_h \approx \underbrace{\left(\int_0^{\infty} dE\, e^{-\beta E} D_c(E+E_c)\right)}_{\equiv N_\varphi^{(e)}} \underbrace{\left(\int_0^{\infty} dE\, e^{-\beta E} D_v(E_v - E)\right)}_{\equiv N_\varphi^{(h)}} e^{-\beta E_g}$$

$E_g = E_c - E_v$

(independent of $\mu$!)

$$\boxed{N_e N_h \approx N_\varphi^{(e)} N_\varphi^{(h)} e^{-\frac{E_g}{k_B T}}}$$

"mass-action eqn"

Since $N_e = N_h$ in a pure semiconductor (each electron in cond. band corresponds to a hole in the valence band) get:

$$N_e \approx \sqrt{N_\varphi^{(e)} N_\varphi^{(h)}}\, e^{-\frac{E_g}{2k_BT}}$$ , if cond. and val. bands are equal: $N_\varphi^{(e)} = N_\varphi^{(h)}$

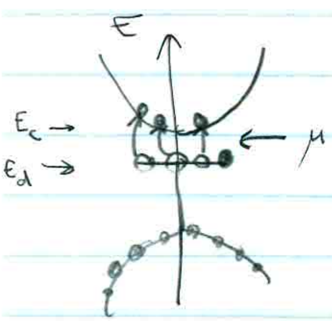
$$\approx N_\varphi\, e^{-\frac{E_g}{2k_BT}}$$

Compare this to the eqn. for $N_e$: $N_e = \underbrace{\int_0^\infty d\epsilon\, e^{-\beta\epsilon} D_c(\epsilon+\epsilon_c)}_{N_\varphi} \times e^{\beta(\mu-\epsilon_c)}$ :

$$\Rightarrow \frac{\mu-\epsilon_c}{k_BT} = -\frac{E_g}{2k_BT} \Rightarrow \boxed{\mu = -\frac{E_g}{2} + E_c}$$ $\mu$ (or $E_F$) is at mid-gap!

$\Rightarrow$ If effective masses $m_c$ and $m_v$ are equal, the Fermi level is at midgap!

Chemical potential for doped semiconductors




Donor case

$$N_e = \int_0^\infty d\epsilon \frac{1}{e^{\beta(\epsilon+\epsilon_c-\mu)}+1} D_c(\epsilon+\epsilon_c)$$

$$N_h = \underset{\substack{\uparrow \\ \#\text{ of donors}}}{N_D} \left(1 - \frac{1}{e^{\beta(\epsilon_d-\mu)}+1}\right) = N_D \frac{1}{e^{-\beta(\epsilon_d-\mu)}+1}$$

Assume $|\epsilon_c - \mu| \gg k_BT$, (e.g. $T\to 0$) but $|E_d - \mu| \sim k_BT$ :

$$N_e \approx \underbrace{\left(\int_0^\infty d\epsilon\, e^{-\beta\epsilon} D_c(\epsilon+\epsilon_c)\right)}_{N_\varphi^{(e)}} e^{-\beta\epsilon_c} e^{\beta\mu}$$

$$N_h = \underbrace{N_d \left(1 - \frac{1}{e^{\beta(\mu-\epsilon_d)}+1}\right)}_{N_d - N_h = \# \ldots\ N^{(0)}} e^{-\beta(\mu-\epsilon_d)} = N_d^{(0)} e^{\beta\epsilon_d} e^{-\beta\mu}$$

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$$\Rightarrow N_e N_h = N_d^{(0)} e^{\beta E_d} \cancel{e^{-\beta\mu}} N_\phi^{(e)} e^{-\beta E_c} \cancel{e^{\beta\mu}} = N_d^{(0)} N_\phi^{(e)} e^{-\beta(E_c - E_d)}$$

$$\boxed{\frac{N_e N_h}{N_d^{(0)}} = N_\phi^{(e)} e^{-\beta(E_c - E_d)}}$$

Mass action law for electrons, holes, and neutral donors (3 species!)

Since $N_h = N_e$ and $N_d^{(0)} = (N_D - N_e)$ we can write:

$$\frac{N_e^2}{N_D - N_e} = N_\phi^{(e)} e^{-\beta(E_c - E_d)} \Rightarrow \boxed{N_e^2 + \left(N_\phi^{(e)} e^{-\beta(E_c - E_d)}\right) N_e - N_d N_\phi^{(e)} e^{-\beta(E_c - E_d)} = 0}$$

Solve for $N_e$:

$$N_e = \frac{1}{2}\left\{ -N_\phi^{(e)} e^{-\beta(E_c - E_d)} \pm \sqrt{\left(N_\phi^{(e)} e^{-\beta(E_c - E_d)}\right)^2 + 4 N_d N_\phi^{(e)} e^{-\beta(E_c - E_d)}} \right\}$$

Pick $+$:

$$\boxed{N_e = \frac{1}{2} N_\phi^{(e)} e^{-\beta(E_c - E_d)} \left\{ \sqrt{1 + \underbrace{\frac{4N_d}{N_\phi^{(e)}} e^{\beta(E_c - E_d)}}} - 1 \right\}}$$

When $T \to 0$, This term dominates:

$$N_e \approx \frac{1}{\cancel{2}} N_\phi^{(e)} e^{-\beta(E_c - E_d)} \sqrt{\frac{\cancel{4} N_d}{N_\phi^{(e)}} e^{\beta(E_c - E_d)}} = \sqrt{N_\phi^{(e)} N_d}\, e^{-\beta\frac{(E_c - E_d)}{2}}$$

$$= N_\phi^{(e)} e^{-\beta E_c} e^{\beta\mu}$$

$$\Rightarrow \sqrt{\frac{N_d}{N_\phi^{(e)}}} = e^{-\beta E_c + \beta\mu + \frac{\beta}{2}E_c - \frac{\beta E_d}{2}} \Rightarrow \ln\left(\sqrt{\frac{N_d}{N_\phi^{(e)}}}\right) = -\frac{\beta}{2}(E_c + E_d) + \beta\mu$$

$$\Rightarrow \mu(\text{small } T) = \left(\frac{E_c + E_d}{2}\right) + k_B T \ln\underbrace{\left(\sqrt{\frac{N_d}{N_\phi^{(e)}}}\right)}_{>1 \text{ b.c. } N_d > N_\phi^{(e)}!} \gtrsim \left(\frac{E_c + E_d}{2}\right).$$