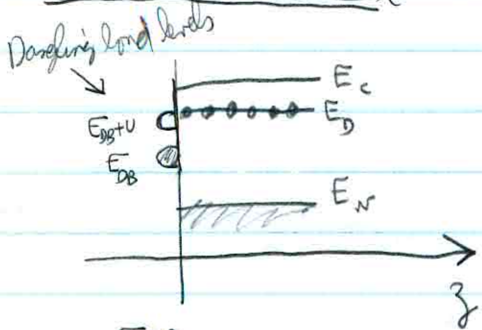
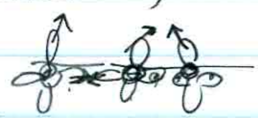
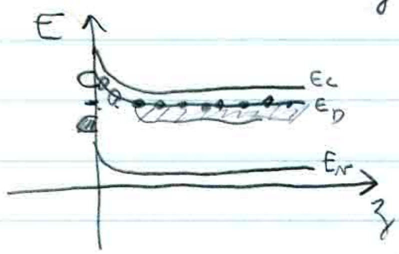
Lecture #10: Quantum confinement

Semiconductor surface (Semicond + vacuum)

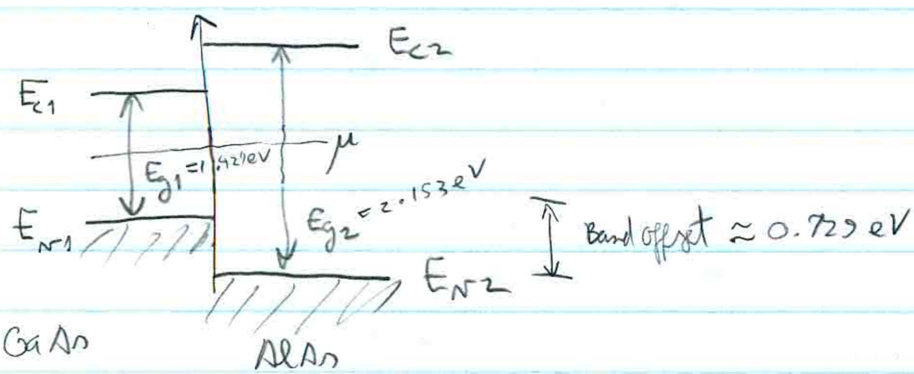




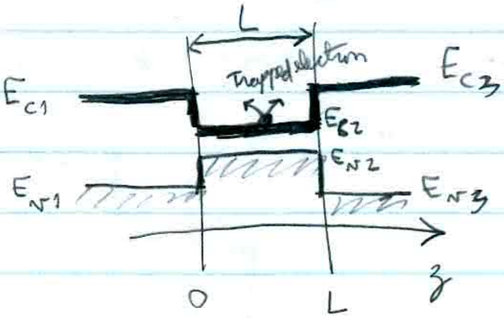


Just like the Schottky barrier.

Two pure (undoped) semiconductors with different gap

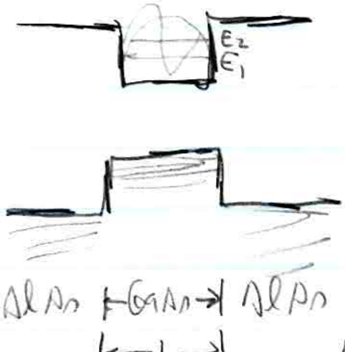


Quantum Confinement



"Quantum well".

Dope AlAs with donors to get e^-'s in QW.



$$N\frac{\lambda}{2} = L \Rightarrow k_N = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{2L}{N}} = N\frac{\pi}{L}$$

$$\boxed{E_N = \frac{\hbar^2 k_N^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 N^2}$$

Criteria for electrons to occupy only the first level: $(E_2 - E_1) \gg k_B T$

Pick $10 k_B T$:

$$|\Delta E| = E_2 - E_1 = \frac{3}{2}\frac{\hbar^2 \pi^2}{m L^2} \sim 10\, k_B T$$

$$\Rightarrow L \sim \sqrt{\frac{3\hbar^2 \pi^2}{20 m k_B T}}$$

$\Rightarrow$ For $T = 300K$, $L \sim$ few nanometres!

When $(E_2 - E_1) \gg k_B T$, the z coordinate of the electrons is "frozen out"

and the electrons are effectively 2 dimensional!

Bands are given by $\boxed{E_{\vec{k},N} = E_N + \frac{\hbar^2 k_\perp^2}{2m}}$ ("subbands")

Density of states:

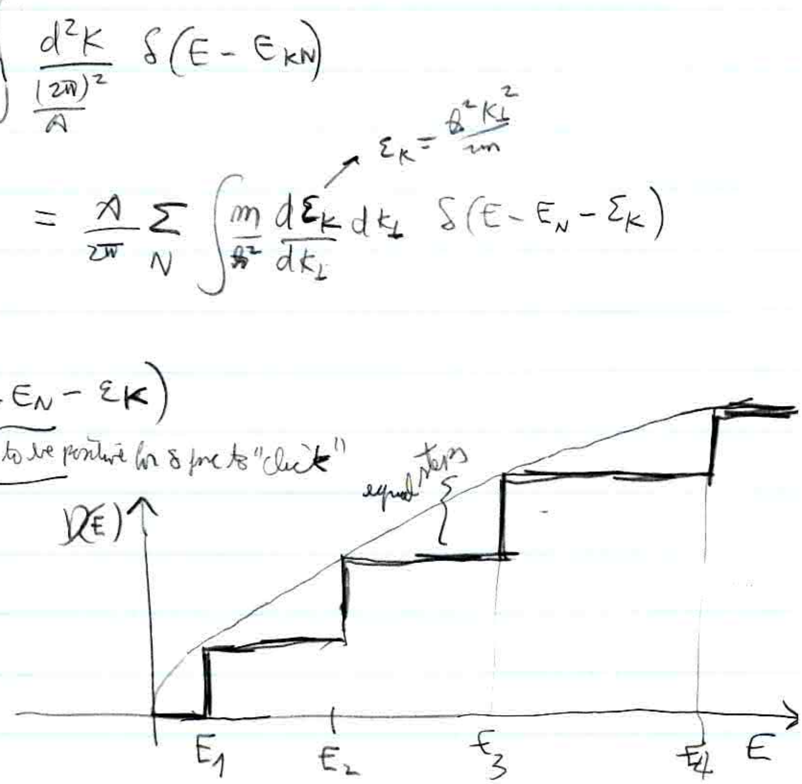
$$D(E) = \sum_{k,N} \delta\left(E - E_{\vec{k},N}\right) = \sum_N \int \frac{d^2 k}{\frac{(2\pi)^2}{A}} \delta(E - E_{kN})$$

$$= \sum_N \int \frac{2\pi k_\perp dk_\perp}{\frac{(2\pi)^2}{A}} \delta(E - E_{kN}) = \frac{A}{2\pi}\sum_N \int \frac{m}{\hbar^2}\frac{d\varepsilon_k}{dk_\perp} dk_\perp\, \delta(E - E_N - \varepsilon_k)$$

$\varepsilon_k = \frac{\hbar^2 k_\perp^2}{2m}$

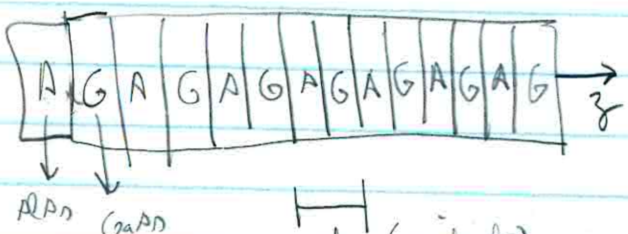
$$= \frac{mA}{2\pi\hbar^2}\sum_N \underbrace{\int_0^\infty d\varepsilon_k\, \delta(\underbrace{E - E_N}_{\text{has to be positive for } \delta \text{ fnc to "click"}} - \varepsilon_k)}_{\Theta(E - E_N)}$$

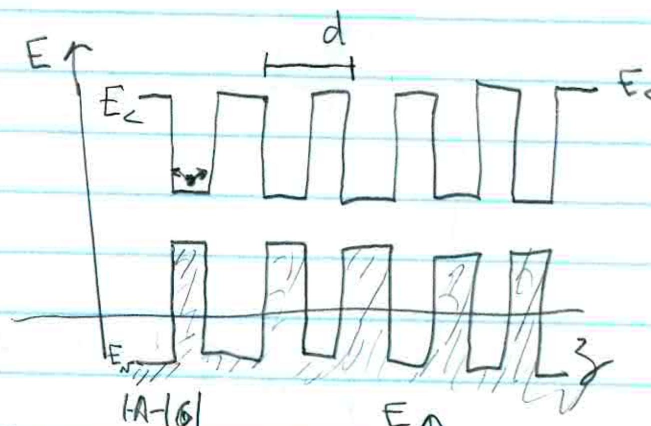
$$D(E) = \frac{mA}{2\pi\hbar^2}\sum_N \Theta(E - E_N)$$

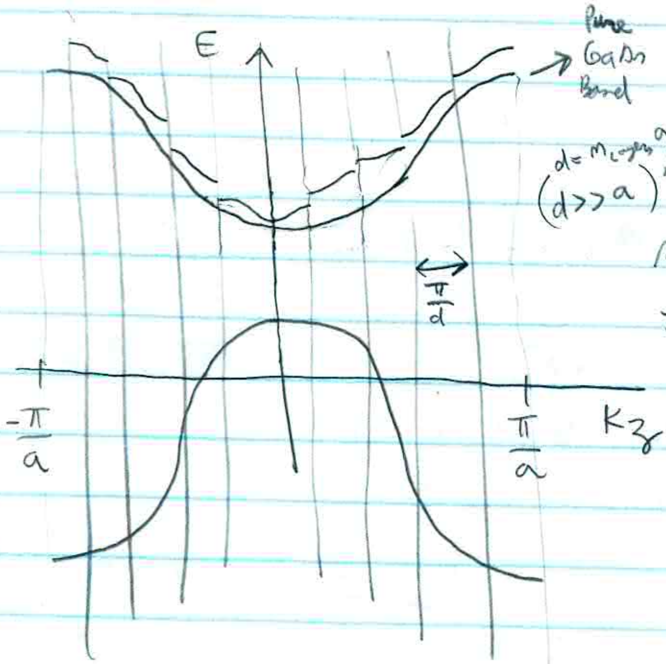


Dimension	$D(E)$	Name of the artificial structure
3	$\propto \sqrt{E}$	"Bulk"
2	$\propto \sum_N \Theta(E - E_N)$	"Quantum well"
1	$\propto \sum_N \frac{\Theta(E-E_N)}{\sqrt{E - E_N}}$	"Quantum wire"
0	$\propto \sum_N \delta(E - E_N)$	"Quantum dot"

Superlattices

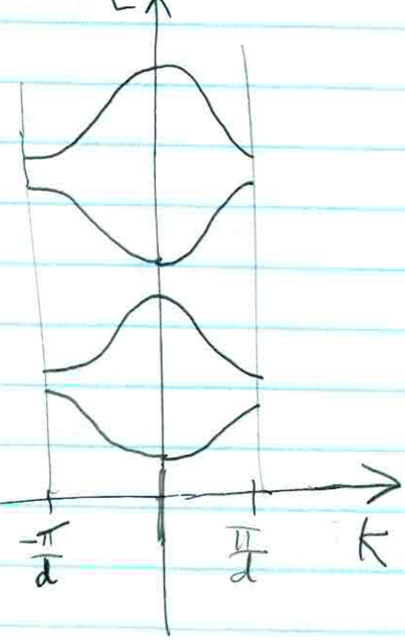




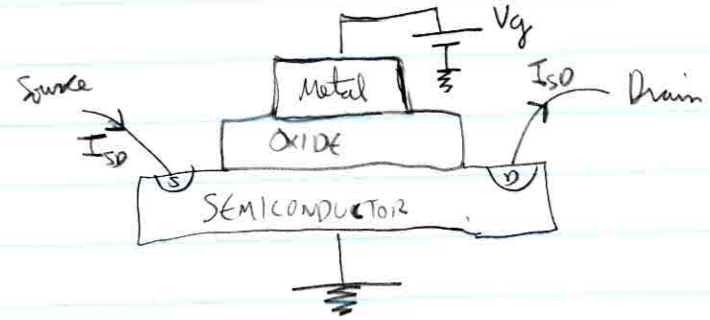


reduced zone

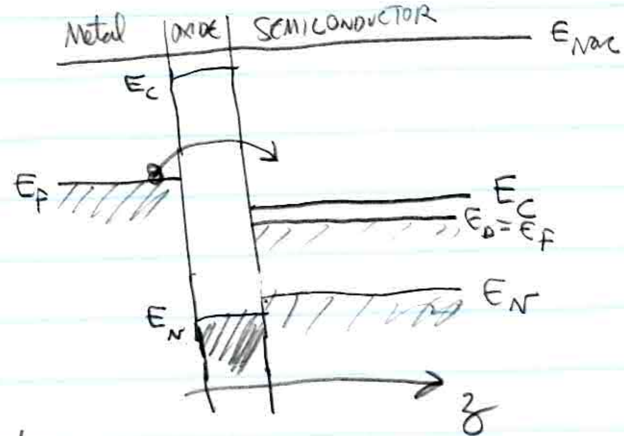
$\Rightarrow$

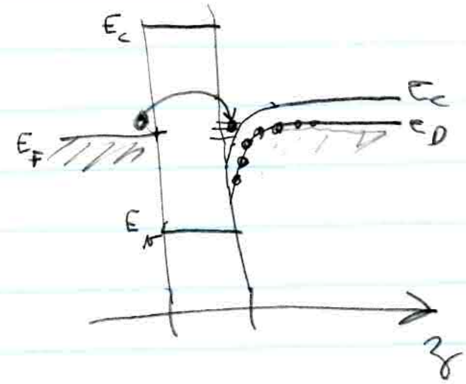


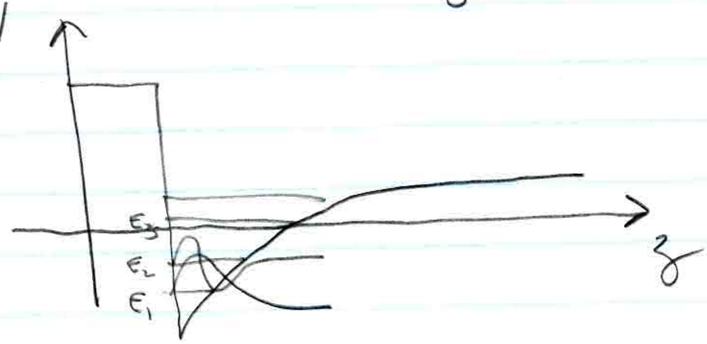
MOSFET (METAL OXIDE SEMICONDUCTOR FIELD EFFECT TRANSISTOR)



Flat bands:







"Triangular well"

Gate voltage V_g controls the Fermi level in the metal

$\Rightarrow$ V_g controls 2DEG electron density! Hence V_g controls the resistance of the device $\Rightarrow$ $\begin{cases} V_g \text{ high} \Rightarrow n_{2DEG} \text{ high} \Rightarrow I_{SD} \text{ high} \\ V_g \text{ low} \Rightarrow n_{2DEG} \sim 0 \Rightarrow I_{SD} \sim 0 \end{cases}$

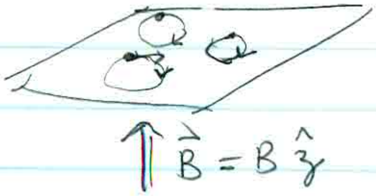
Signal applied to V_g gets amplified in I_{SD} $\Rightarrow$ TRANSISTOR (works as a microphone!)

$\Rightarrow$ MOSFET IS The basis of the electronics industry

$\Rightarrow$ MOST abundant artificial object!!

Landau levels in a 2DEG

Very important system $\Rightarrow$ Quantum hall effect, "Standard of resistance" (The von Klitzing).



$\vec{B} = B\hat{z}$

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad \vec{A} = Bx\,\hat{y} \qquad (\text{"assymmetric gauge"})$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \partial_x \hat{x}\times(Bx\hat{y}) = \partial_x(Bx)\,\hat{x}\times\hat{y} = B\hat{z} \checkmark$$

$$\mathcal{H} = \frac{(\vec{p} - q\vec{A})^2}{2m}$$

Note: $\vec{p} = \frac{\hbar}{i}\vec{\nabla}$ is "canonical momentum"

$\hbar\vec{k} = (\vec{p} - q\vec{A})$ is "kinetic momentum" (related to velocity!)

$$\mathcal{H} = \frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - qBx\hat{y}\right)^2\psi = \frac{1}{2m}\left\{-\hbar^2\partial_x^2 + \left(\frac{\hbar}{i}\partial_y - qBx\right)^2 - \hbar^2\partial_z^2\right\}$$

$$= \left\{-\frac{\hbar^2}{2m}(\partial_x^2 + \partial_z^2) - \frac{\hbar^2}{2m}\partial_y^2 - 2\frac{qB\hbar}{i\,2m}x\,\partial_y + \frac{(qBx)^2}{2m}\right\}$$

$$= \left\{-\frac{\hbar^2}{2m}\nabla^2 + i\,\frac{qB\hbar}{m}x\partial_y + \frac{1}{2}m\left(\frac{qB}{m}\right)^2 x^2\right\}$$

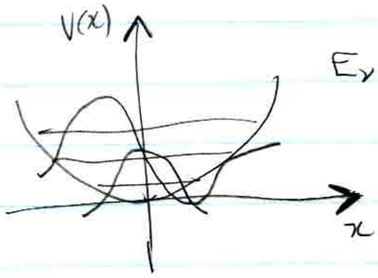
Guess: $\psi(x,y,z) = e^{i(k_y y + k_z z)}\phi(x)$

$$\mathcal{H}\psi = \left[\frac{\hbar^2}{2m}(k_y^2 + k_z^2) - \frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} - \underbrace{\hbar\frac{qB}{m}}_{\hbar\omega_c}x k_y + \frac{1}{2}m\underbrace{\left(\frac{qB}{m}\right)^2}_{\omega_c^2}x^2\right]\cancel{e^{i(k_y y + k_z z)}}\phi = E\,\cancel{e^{i(k_y y + k_z z)}}\phi$$

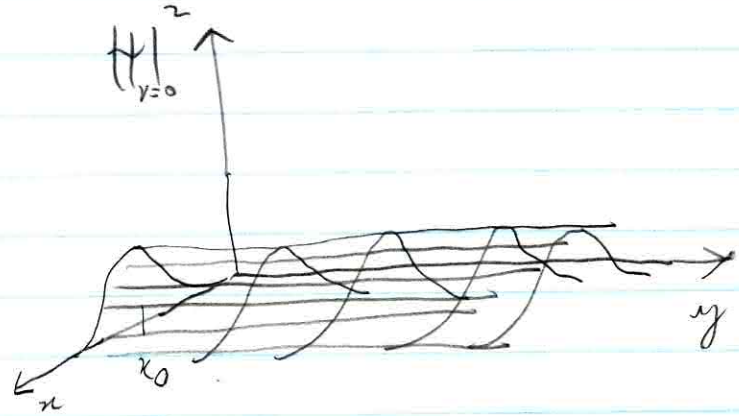
$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + \frac{1}{2}m\omega_c^2\Big(x - \underbrace{\frac{\hbar k_y}{m\omega_c}}_{x_0}\Big)^2\right]\phi(x) = \left(E - \frac{\hbar^2 k_z^2}{2m}\right)\phi(x)$$

$$x_0 = \frac{\hbar k_y}{m\omega_c} \qquad \omega_c = \frac{qB}{m}$$

$\Rightarrow$ Harmonic oscillator with natural frequency ω_c in 1D (x coordinate!)



$$E_\nu = \left(\nu + \tfrac{1}{2}\right)\hbar\omega_c$$



For $\nu = 0$, ground state is:

$$\psi(\vec{r}) = \frac{e^{i\frac{m\omega_c}{\hbar}x_0}\, e^{ik_z z}}{\sqrt{A}} \; \frac{e^{-\frac{1}{2}\left(\frac{x}{l_B}\right)^2}}{\sqrt{2\pi l_B^2}}$$

Check this at home:

$$l_B = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar}{m\frac{|q|B}{m}}} = \sqrt{\frac{\hbar}{|q|}}\,\frac{1}{\sqrt{B}} \qquad \text{"Magnetic length".}$$

$B \sim 1\,T$, $q = -e$ $\Rightarrow l_B = \sqrt{\frac{1.05\times10^{-34}\,Js}{1.6\times10^{-19}\,C \times 1\,T}} = 2.5\times10^{-8}\,m = 25\,nm$

# of states that fit into a Landau level:

From periodic boundary conds: $e^{ik_y(y+L_y)} = e^{ik_y y} \Rightarrow e^{ik_y L_y} = 1 \Rightarrow k_y = \frac{2\pi}{L_y} N_y$ (one k_y per $\frac{2\pi}{L_y}$)

From $x_0 = \frac{\hbar k_y}{m\omega_c}$:

If x_0 runs from $-\frac{L_x}{2}$ to $\frac{L_x}{2}$, then $\Rightarrow -\frac{m\omega_c L_x}{2\hbar} \le k_y \le \frac{m\omega_c L_x}{2\hbar}$

of k_y's available: $N = \dfrac{\frac{m\omega_c L_x}{\hbar}}{\frac{2\pi}{L_y}} = \dfrac{m\omega_c}{2\pi\hbar} L_x L_y = \dfrac{qB}{h} A$

($A = L_x L_y$ is the area).