Lecture #20  Average electron phonon scattering times, electron - photon interactions

Average time for phonon emission or absorption by an electron with wavevector $\mathbf{q}$:

$\frac{1}{\tau} = \frac{2\pi}{\hbar} \alpha_2 \sum_{k} |g_{k}^{-1} \Gamma_{k} \mathbf{v}_{k} |^2 S(E_{k} - E_{e})$

$= \frac{2\pi}{\hbar} \sum_{k} \frac{D_{k}^2 \mathbf{v}_{k}^2}{2eV_{N}} \left| \langle N_{k}+1 | a_{k}^{+} c_{q-k}^{+} c_{q-K} | N_{k} \rangle \right|^2 S\left(\frac{E_{q-K} - E_{k}}{E_{q-K}}\right) S\left(\frac{E_{q-K} - E_{k}}{E_{q-K}}\right)$

$+ \frac{2\pi}{\hbar} \sum_{k} \frac{D_{k}^2 \mathbf{v}_{k}^2}{2eV_{N}} \left| \langle N_{k}-1 | a_{k}^{+} c_{q-K}^{+} c_{q} | N_{k} \rangle \right|^2 S\left(\frac{E_{q-K} - (E_{k} + 2\hbar \omega_{k})}{E_{q-K}}\right) S\left(\frac{E_{q-K} - (E_{k} + 2\hbar \omega_{k})}{E_{q-K}}\right)$

$\frac{1}{\tau} = \frac{2\pi}{\hbar} \frac{D_{k}^2 \mathbf{v}_{k}^2}{2eV_{N}} \int \frac{d^3k}{(2\pi)^3} \left[ N_{k} (1 - N_{q-k}) (N_{k+1}) S\left(\frac{E_{q-k} + 2\hbar \omega_{k} - E_{q}}{E_{q-k}}\right) + N_{k} (1 - N_{q+k}) N_{k} S\left(\frac{E_{q+k} - E_{q} - 2\hbar \omega_{k}}{E_{q-k}}\right) \right]$

Assume high temperature: $N_{q}$, $N_{q-k}$ << 1

Approximately: $\frac{\mathbf{q}^2 - K^2}{\hbar^2} \gg 2\hbar \omega_{k}$ (high electron density)

$E_{q+k} = (3\hbar \omega_{k} - E_{q} \approx \frac{\hbar^2 q^2}{m} (K^2 + \mathbf{q}^2) + \frac{\hbar^2 q^2}{m} \approx \frac{1}{2} m \mathbf{q}^2 + \mathbf{k}^2 \gamma_{0} \frac{\hbar^2}{m} + \frac{\hbar^2}{m} \mathbf{k}^2$
\[ \frac{1}{N_k} \approx \frac{D^2}{8\pi^2} \frac{1}{8\pi^2} \int_0^\infty d\omega d\omega' \int d^3k d^3k' \left( 1 + N_k \right) \delta \left( \frac{\omega^2}{m^2} + \frac{k^2}{m^2} + \frac{k'^2}{m^2} - 2q^2 \right) \]

\[ = \frac{D^2}{4\pi^2} \int_0^\infty d\omega \left( 1 + N_k \right) \frac{1}{\sqrt{g}} \int_0^1 d\omega' \delta \left( \omega + \frac{k}{2q} \right) \]

\[ = \frac{D^2}{4\pi^2} \left( \frac{m}{\sqrt{g}} \right) \int_0^{2g} dk k^2 \left( 1 + 2N_k \right) \]

\[ N_k = \frac{1}{k_BT} \approx \frac{k_BT}{\Delta E_k} \gg 1 \]

\[ \frac{1}{N_k} \approx \frac{D^2}{8\pi^2} \frac{m}{\sqrt{g^2}} \frac{2^2}{\sqrt{2}} \frac{k_BT}{\Delta E_k} \]

\[ \langle \left\langle \frac{1}{N_k} \right\rangle \rangle = \frac{D^2}{4\pi^2} \frac{m k_BT}{\Delta E_k} \]

\[ \approx \frac{D^2}{4\pi^2} \frac{m k_BT}{\Delta E_k} \frac{2^2}{3} \]

\[ \langle \left\langle \frac{1}{N_k} \right\rangle \rangle \approx \frac{D^2}{4\pi^2} \frac{m k_BT}{\Delta E_k} \frac{2^2}{3} \]

\[ \Rightarrow \text{Typical solid: } \langle E \rangle \approx \frac{5e^2}{n^2}, \quad N \approx 5 \times 10^{18}, \quad \Delta E \approx 12eV, \quad m \approx m_e, \quad T \approx 300K \]

\[ \Rightarrow \langle n \rangle \approx 10^{18} \]
Electron - photon interaction's

For EM waves in the Coulomb gauge $\nabla \cdot A = 0$ we have $\phi = 0$ and $A \neq 0$. This leads to

the electron's Hamiltonian:

$$H = \frac{1}{2m} \left( \hat{\mathbf{p}} - q \hat{\mathbf{A}} \right)^2 = \frac{\hat{\mathbf{p}}^2}{2m} - \frac{1}{m} \left( \hat{A} \cdot \hat{\mathbf{p}} + \hat{\mathbf{p}} \cdot \hat{A} \right) + \frac{1}{2m} q^2 \hat{A}^2$$

$\nabla \cdot A = 0$

$\Rightarrow \hat{\mathbf{p}} \cdot A = A \cdot \hat{\mathbf{p}}$

$\Rightarrow H_{e-photon} = -\frac{e}{m} \hat{A} \cdot \hat{\mathbf{p}}$

The electron wavefunction:

$$\hat{\psi}(\hat{\mathbf{r}}, t) = \sum \frac{\hat{\psi}_{k_1 k_2}}{k_{12}} \sqrt{\frac{\theta}{2\pi \hbar}} \left( \frac{1}{r} \right)^{k_1} e^{i \left( k_1 x - k_2 y + k_3 z \right)}$$

To describe the light, use: (similarly, for light)

$$H_{e-photon} = \sum_{k_1 k_2} \frac{\hat{\psi}_{k_1 k_2} \hat{\mathcal{H}}_{k_1 k_2} \hat{\psi}_{k_1 k_2}^*}{\sqrt{2\pi \hbar}}$$

$$= -\frac{e^2}{m} \sum_{k_1 k_2} \frac{1}{\sqrt{2\pi \hbar}} \left[ \alpha_{k_1 k_2} \frac{\hat{c}^+_{m_{k_1} n_{k_2}}}{\sqrt{\lambda_{m_{k_1} n_{k_2}}}} \left( \frac{1}{\sqrt{2\pi \hbar}} \int d^3r e^{i(k_1 - k_2 - k) \cdot \hat{\mathbf{r}}} \right) \hat{\mathcal{H}}_{k_1 k_2} \hat{\mathcal{H}}_{k_1 k_2}^* \right]$$

Again assume $K << (V_{cell})^2$ so that $\lambda_{m_{k_1} n_{k_2}}(\hat{\mathbf{r}})$.
\[ H_{m'm} = -\frac{e}{m} \sum_{k} \sum_{k'f} \left( \hat{\mathcal{H}}_{k} \langle m' | \hat{p} | m \rangle \right) \left( \hat{\mathcal{A}}_{k} \hat{\mathcal{A}}_{k'} + \hat{\mathcal{C}}_{m} \hat{\mathcal{C}}_{k} \right) \]

where

\[ \langle m' | \hat{p} | m \rangle = \frac{1}{\text{Vol}} \int d^3\mathbf{r} \; \mathcal{M}_{m'}^{*}(\mathbf{r}) \left( \hat{\mathcal{A}}_{m} \right) \mathcal{M}_{m}(\mathbf{r}) \]

**Intertwined transitions in a semiconductor**

\[ \frac{1}{2} = \frac{1}{N_{k}} \sum_{k'f} \frac{\pi}{2 m} \frac{e^{2}}{\varepsilon V \omega m} \sqrt{\langle n' | \hat{p} | n \rangle \cdot \hat{n}} N_{k} N_{k'} \left( 1 - N_{k'c} \right) \varepsilon \left( E - E_{c} - \frac{1}{2} \omega \right) \]

and \( \mathbf{k}' + \mathbf{k} = \mathbf{k} \).

\[ E_{k'} = E_{gap} + \frac{\hbar^{2}}{2 m_{e}} k_{c}^{2} \]

\[ E_{k} = -\frac{\hbar^{2}}{2 m_{k}} k_{c}^{2} \]

\[ (E_{k'} - E_{k}) = E_{gap} + \frac{\hbar^{2}}{2 m_{e}} k_{c}^{2} + \frac{\hbar^{2}}{2 m_{c}} \left( k_{c} - k \right)^{2} \left( k_{c} \right) \]

\[ \sim E_{gap} + \frac{\hbar^{2}}{2} \left( \frac{1}{m_{e}} + \frac{1}{m_{c}} \right) k_{c}^{2} \]

\[ \frac{1}{m_{c}} \rightarrow \text{electron reduced mass} \]
\[ \frac{1}{Z} = \frac{2\pi}{9} \int \frac{d^3k}{(2\pi)^3} \frac{\hbar e^2}{2\varepsilon_{\nu} \hbar \omega} \left( \langle n|\prod_{\nu}^{p}|\pi \rangle \cdot \hat{n} \right)^2 \delta \left( \varepsilon_{\omega} - (\varepsilon_{\gamma \nu} + \frac{k^2}{m_n}) \right) \]

\[ \delta \left( \varepsilon_{\omega} - (\varepsilon_{\gamma \nu} + \frac{k^2}{m_n}) \right) = \frac{\delta (k-k_0)}{\Gamma'}(\text{where } \Gamma') \]

\[ \frac{\hbar e^2}{2\varepsilon_{\nu} \hbar \omega} \left( \langle n|\prod_{\nu}^{p}|\pi \rangle \cdot \hat{n} \right)^2 \left( \frac{V}{(2\pi)^3} \frac{m_n}{\hbar^2} \sqrt{\frac{2m_n}{\hbar^2} (\varepsilon_{\omega} - \varepsilon_{\gamma \nu})} \right) \]

\[ D_{\text{exciton}} (\varepsilon_{\omega} - \varepsilon_{\gamma \nu}) \]

Valid in \( \varepsilon_{\omega} \gg \varepsilon_{\gamma \nu} \), otherwise function of \( \omega \) and \( \text{modulation} \) need to be reconsidered.

**Alternative form of e-photon interaction:**

From the \( \bar{\eta} \cdot \bar{p} \) coupling we get:

\[ H_{m'm} = -\frac{\hbar}{m} \sum_{k} \sqrt{\frac{5}{2\varepsilon_{\nu} \hbar k_1}} \left( \bar{\eta}_{k1} \cdot \langle m'|\prod_{\nu}^{p}|m \rangle \right) \left[ \hat{a}^+_m \hat{c}_{m'k} + \hat{c}^+_m \hat{a}_{m'k} e^{-i\omega t} \right] \]

\[ + \hat{a}^+_m \hat{c}^+_m \hat{c}^+_{m'k} e^{-i\omega t} \]

Before we took \( t=0 \).
Plug \( \hat{p} = m \frac{\partial}{\partial t} \hat{x} = \pm m i \omega \hat{x} \):

\[
\hat{H}_{m,n} = i e \sum \sum \sqrt{\frac{\omega_{mn}}{2\epsilon V}} (\hat{\alpha}_{m,n} \hat{\beta}_{m,n}) \left[ \hat{a}_{k_1} \hat{a}^+_{k_2} \right. \\
\left. + \hat{a}_{k_2} \hat{a}^+_{k_1} \right] e^{-i\omega t}
\]

"Electric dipole" equivalent to \( \hat{A} \cdot \hat{p} \) interaction.

Formally, one may derive this from multiple expansion instead of \( \hat{A} \cdot \hat{p} \).